MINISTÈRE
DE L'ÉCONOMIE,
DES FINANCES
ET DE LA SOUVERAINETÉ
INDUSTRIELLE ET NUMÉRIQUE
*Liberté*
*Égalité*
*Fraternité*

Tresor-Economics

N° 334 • October 2023

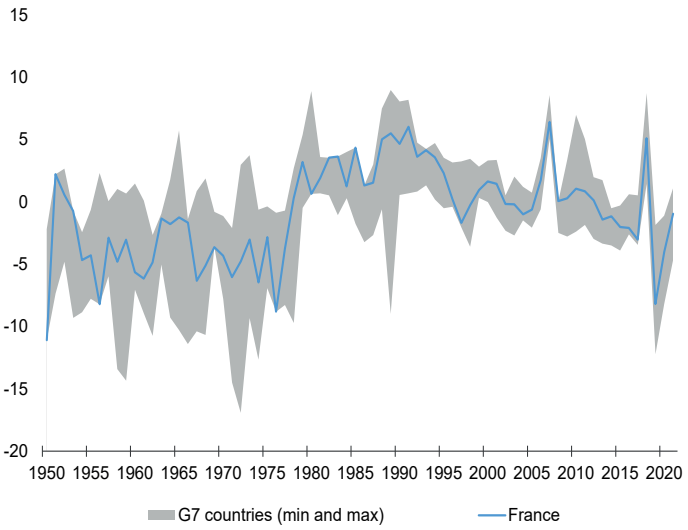
Direction générale du Trésor

# Interest Rates, Growth and Public Debt Sustainability

*Guillaume Clavères*

- The sustainability of public debt depends on its long-term trajectory. This trajectory depends in turn on fiscal policies (i.e. the accumulation of annual primary balances) and the differential between the interest rate (*r*) and the growth rate of GDP (*g*).
- If the primary balance is zero, the ratio of debt as a percentage of GDP increases, if the interest rate is greater than the growth rate (*r*–*g*>0) and it decreases in the opposite case. In the case of a primary deficit, the effect is more ambiguous: a positive *r*–*g* differential accelerates the increase in the debt ratio, while a negative differential contains the increase in the ratio and may even reduce it in some cases.
- For France, and other major advanced countries, the interest rate-growth differential has been very volatile and positive over long periods (see Chart). Since the end of the 1990s, the interest rate-growth differential has narrowed for structural reasons, notably with excess savings at the global level which lowered risk-free interest rates, and even became negative in the past decade.
- As measured by nominal borrowing rates, the differential could turn positive again in certain advanced economies as soon as 2023-2024, given the factors impeding growth and the surge in interest rates. As measured by the implicit interest rate, meaning the average cost of debt, the differential should remain negative in the medium term.
- Caution is called for when using the observed interest rate-growth differential as a fiscal policy indicator, since it is impossible to foresee its future values. Furthermore, a negative interest rate-growth differential is not, generally speaking, sufficient to contain government debt in the presence of a primary deficit.
- History shows that the link is not one-way, because the interest rate-growth differential is affected by the debt ratio: the greater the increase in the debt ratio, the greater the increase in the *r*–*g* differential.

**Differential between nominal 10-year interest rates and nominal growth rates (percentage points)**




*Sources: DG Trésor calculations. Jordà-Schularick-Taylor Macrohistory Database, OECD, IMF latest data point: 2021, 2022-2023 forecasts.*

How to read this chart: In 1970, the *r*–*g* differentials of the G7 countries ranged from –10.7 percentage points to +1.9 percentage points France's differential stood at –5.1 percentage points.

# 1. The interest rate-growth differential, along with the primary balance, determines public debt dynamics

## 1.1 The accumulation of public debt and the *r–g* differential

The borrowing rates paid by advanced countries' governments have risen since summer 2021, ending a long period of low financing costs. On top of this recent change in the macroeconomic environment, uncertainty about global growth has emerged. This context has affected the differential between the interest rate on public debt (*r*) and the growth rate of GDP (*g*), usually labelled *r–g*. This differential is used to compare the cost of debt with the increase in the available resources for covering debt service.In accounting terms, public debt dynamics depend on:

- the differential between the financing cost, captured by the interest rate, and GDP growth (public debt is generally measured as a percentage of GDP since GDP is a synthetic measure of the country's tax base)
- the ratio of the primary balance to GDP, meaning the government balance, excluding interest payments
- and stock-flow adjustments that may contribute to a change in the debt ratio on a more temporary basis, resulting from financial transactions that are not recognised as part of the deficit, such as an acquisition of financial assets by the public sector.

There are different ways to measure the interest rate on public debt and they provide complementary information. The implicit interest rate calculates the cost of interest as a percentage of the debt. This rate is very frequently used and it is a relevant indicator for the accumulation of government debt (see Box 1).

It provides a view of the average funding cost for the entire government debt stock. It is the result of past borrowing rates, but it provides less information about contemporary financing conditions. The implicit rate is more stable than the market rates and its use attenuates variations over short periods. At the same time, long-term sovereign yields represent the financing conditions that governments actually face for their debt issuance. These yields factor in expectations about future short-term rates, term premiums and, potentially, risk premiums. Furthermore, long-term rates are fairly homogenous indicators from one country to the next and can be used for international comparisons.

Much of the literature focuses on nominal rates. At first glance, the *r–g* differential is the same whether measured in real terms or nominal terms, since inflation is subtracted from both rates at the same time. This means the differential is the same. However, the differential between *r* and *g* may be sensitive to inflation, if inflation affects *r* and *g* endogenously and not mechanically. At first, inflation may boost nominal growth and lift *g* in nominal terms, which decreases the *r–g* differential until nominal interest rates also rise. If inflation persists, the rise in interest rates will eventually be transmitted to long-term rates and implicit rates. This will increase the *r–g* differential, especially if inflation is the result of a supply-side shock that hampers growth. On the *r* side, an inflation shock or increased uncertainty about inflation may increase *r* in real terms.[1] In addition, persistently tight monetary policy to fight inflation may end up weakening growth.

(1) See, for example, M. K. Brunnermeier, S. Merkel & Y. Sannikov (2020), "Debt as Safe asset: Mining the Bubble", Princeton University.

## Box 1: Law of motion for public debt

The debt ratio as a percentage of GDP moves as follows:

$$d_t = d_{t-1} \times \frac{(1 + r_t)}{(1 + g_t)} - pgb_t$$

where $d_t$ represents the government debt ratio as a percentage GDP at the end of year *t*

$r_t$ represents the implicit interest rate on debt (interest payments on the debt divided by the debt stock at the end of year *t* –1)

$g_t$ represents the nominal GDP growth rate in year *t* compared to the year *t* –1

$pgb_t$ represents the primary government balance as a percentage of GDP in year *t*

The equation can be approximated at the first order by:

$$d_t = d_{t-1} \approx d_{t-1} \times (1 + r_t) - pgb_t$$

Here, $d_{t-1}$ x $(r_t - g_t)$ represents the "snowball effect", meaning the effect of the previous year's debt ratio on the current year's ratio through interest payments and the growth rate. This effect can be positive, when *r*>*g*, or negative, when *r*<*g*.

The above formula can be used to calculate the debt-stabilising primary balance $pgbstab_t$:

$$pgbstab_t = d_{t-1} \times (r_t - g_t)$$

The debt-stabilising primary balance is positive if *r*>*g*, negative if *r*<*g* and zero if *r* = *g*. It also depends on the size of the initial debt stock. The greater the initial debt stock, the greater the debt-stabilising primary government balance will be in absolute terms (large surplus if *r*>*g* and large deficit if *r*<*g*). If the primary deficit in year t is deeper than this debt-stabilising primary deficit, meaning ($pb_t$ < $pbstab_t$), then the debt stock will grow, even if *r*<*g*.

### 1.2 What a negative *r*–*g* differential means for changes in the debt ratio

For a given primary government balance, the larger the interest rate-growth differential (*r*–*g*), the more the debt ratio will increase. When the interest rate is lower than the growth rate, the differential tends to reduce the debt ratio.

There has been intense debate about the role that the *r*–*g* differential plays in fiscal policy-making. O. Blanchard (2019)[2] stated that the narrowing of the differentials between interest rates and growth rates creates fiscal headroom, especially their turning negative in the advanced economies (see Section 2.2). In 2023, Blanchard also stressed that, once inflation returned to its pre-pandemic level, the prevailing macroeconomic context up until 2019, where *r*–*g* differentials were negative, should continue to have a favourable effect on public debt ratios.[3] However, there is a debate about the relevance of the *r*–*g* differential as a tool for fiscal policy management, especially because of the risk of an *r*–*g* reversal (see Section 3.2).

Other research highlights the role played by private capital accumulation and investors' willingness to hold risky financial assets in the government's capacity to rely on public debt and deficits.[4] More generally, this research underlines the issues linked to absorption capacities, market structures, sovereign ratings, investors' credit risk perceptions and investors' preferences.

(2) See O. Blanchard (2019), "Public debt and low interest rates", *American Economic Review*, vol. 109.4, pp. 1197-1229.
(3) See the Peterson Institute for International Economics conference "Summers and Blanchard debate the future of interest rates", 7 March 2023.
(4) See R. Reis (2022), "Debt Revenue and the Sustainability of Public Debt", *Journal of Economic Perspectives*, vol. 36.4, pp. 103-124.

## 2. Differentials between interest rates and growth rates are unstable in the long run

### 2.1 The differentials were persistently negative until the end of the 1970s before rising strongly

Historically, the differential between the 10-year interest rate, which can be used to assess contemporary financial terms in international comparisons, and the growth rate had been negative on average in France and other advanced economies since the end of the nineteenth century. However, it was highly volatile and positive over long periods. This makes it difficult to predict its future value (see Chart on the first page and Tables 1 and 2). In the long run (since 1950), the *r–g* differentials have alternated between three major phases of positive and negative values, with sharp short-term swings, featuring annual variations of several percentage points, and sudden reversals, due in part to unpredictable fluctuations in growth *g*.[5]

Differentials between interest rates and growth rates in the advanced economies went through periods where they were very negative, as was the case between 1950 and 1979. This was attributable to a combination of:

- Strong nominal growth, possibly explained by such factors as the adoption of mass production methods in the United States.[6]
- Low interest rates linked to financial repression policies (control over capital outflows and bank deposit rates), which meant that investors had few alternatives to holding sovereign debt, despite low yields.

The higher differentials between interest rates and growth rates at the end of the 1970s can be explained by disinflation policies that raised interest rates and by slower growth over the period. The interest rate-growth differentials then turned persistently positive for approximately two decades.

**Table 1: Differentials between interest rates and growth rates (nominal 10-year interest rate – nominal growth, 1950-2022)**

(Percentage points)

| | France | Germany | Italy | Canada | United Kingdom | United States | Japan |
|---|---|---|---|---|---|---|---|
| Mean | –1.2 | –0.5 | –1.1 | –1.1 | –1.0 | –1.0 | –2.6 |
| Standard deviation | +4.2 | +3.5 | +5.1 | +4.7 | +3.2 | +3.3 | +5.0 |

*Sources: Jordà-Schularick-Taylor Macrohistory Database, WEO Database (April 2023, latest data point 2021), IMF, Base from Economic Outlook No. 113 (June 2023, latest data point 2021), OECD. DG Trésor calculations.*

**Table 2: Proportion of years where *r>g***

(as a % of years during the period)

| | France | Germany | Italy | Canada | United Kingdom | United States | Japan |
|---|---|---|---|---|---|---|---|
| 1950-2022 | 56 | 57 | 55 | 59 | 62 | 60 | 63 |
| 1950-1979 | 93 | 83 | 100 | 83 | 73 | 83 | 97 |
| 1980-1999 | 5 | 10 | 20 | 5 | 25 | 10 | 30 |
| 2000-2022 | 52 | 65 | 26 | 74 | 78 | 74 | 48 |

*Sources: Jordà-Schularick-Taylor Macrohistory Database, WEO Database (April 2023, latest data point 2021), IMF, Base from Economic Outlook No. 113 (June 2023, latest data point 2021), OECD. DG Trésor calculations.*

How to read this table: France's *r–g* differential was negative 93% of the time between 1950 and 1979.

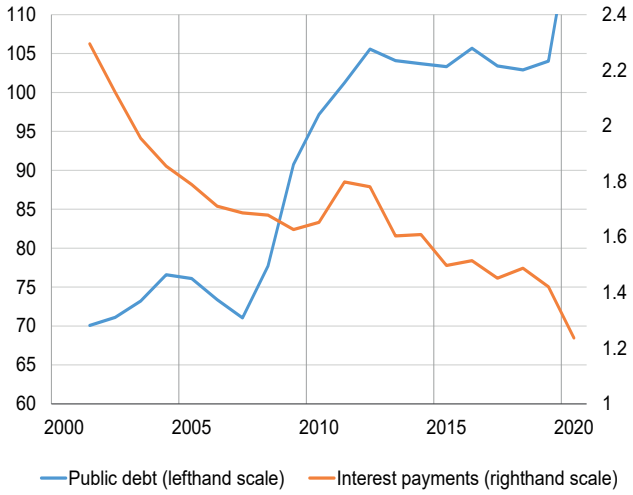
(5) The results would be similar using the implicit interest rate, but less robust for historical international comparisons. Between 2000 and 2022, the signs of both measures and, consequently, the diagnoses, are the same in 89% of the cases, and in 87% of the cases over the period from 1952 to 2022.

(6) For historical analysis, see: B. J. Eichengreen, A. El-Ganainy, R. Esteves & K. J. Mitchener (2021), "In defense of public debt", *Oxford University Press*.

## 2.2 The *r–g* differentials decreased again at the end of the 1990s

Since the end of the 1990s, the interest rate-growth differentials followed a downward trend, as interest rates fell more than nominal growth rates. The downward trend was not the same in every country, with reversals during the financial crisis, but generally speaking, the *r–g* differentials became negative again in the advanced economies in the 2000s. For example, the implicit interest rate (GDP-weighted average rate) fell by 2.6 percentage points from 4.9% between 1998 and 2007 to 2.3% between 2010 and 2019 in the four major economies of the euro area (Germany, France, Italy, Spain). At the same time, average nominal GDP growth fell by 1.6 percentage points from 4.1% between 1998 and 2007 to 2.5% between 2010 and 2019. This meant that the negative snowball effect started to push down debt ratios. The fall in interest rates gradually led to a fall in interest payments, despite the steady increase in debt ratios since the financial crisis (see Chart 1).

**Chart 1: Public debt and interest payments, advanced economies (as a percentage of GDP)**




*Sources: WEO Database (April 2023, latest data point 2019), IMF. DG Trésor calculations.*

Note: The advanced economies include Andorra, Australia, Austria, Belgium, Canada, Croatia, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hong-Kong, Iceland, Ireland, Israel, Italy, Japan, Latvia, Lithuania, Luxemburg, Macao, Malta, Netherlands, New Zeeland, Norway, Porto Rico, Portugal, San Marino, Singapore, Slovakia, Slovenia, South Korea, Spain, Sweden, Switzerland, Taiwan, United Kingdom, United States.

The structural factors that are regularly cited to explain the joint declines in interest, inflation and growth rates in the advanced economies since the end of the 1990s include the global savings glut due to greater risk aversion and the preference for safe assets, weakening productivity, globalisation and ageing populations.[7] All of these factors may help explain the downward trend in the interest rate-growth differential in real terms.[8]

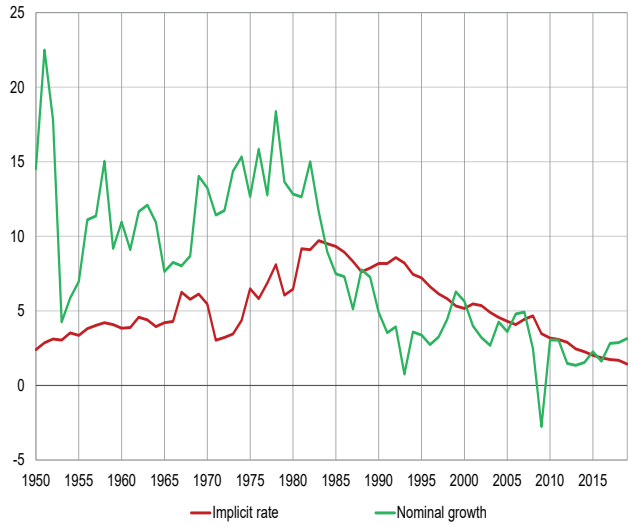
The *r–g* pattern over time is the same in most of the major advanced economies, including France, even when the differential is measured using the implicit interest rate:

- The implicit interest rate on France's public debt was much lower than the nominal growth rate in the 1950s and up until the middle of the 1980s. This meant that the snowball effect tended to lower the debt ratio.
- During the 1980s, interest rates rose to high levels (13.6% in 1980) as policies designed to reduce inflation and attract capital were implemented ("competitive disinflation"). At the same time, the unemployment rate rose to 8.5% in 1985, compared to 5.1% in 1980. High unemployment, combined with high interest rates and restrictive fiscal policies (the "*tournant de la rigueur*" austerity plan) weakened consumption and growth. Consequently, the implicit interest rate on public debt outstripped the growth rate.
- The financial crisis then amplified a trend that had been under way since the end of the 1990s, by dampening inflation expectations, which reduced interest rate expectations and made them less volatile. Quantitative easing policies also reduced risk premiums. Nominal growth finally rose above the interest rate again starting in 2016 (see Chart 2), helping to stabilise the debt ratio until 2019.

(7) A great deal of academic literature analyses these factors, particularly the "secular stagnation" and the forces causing the decline in risk-free interest rates. See, for example, L. Summers (2014), "US economic prospects: Secular stagnation, hysteresis, and the zero lower bound", *Business Economics*, 49.2, pp. 65-73, and A. Jaubertie and L. Shimi (2016), "The debate on secular stagnation: a status report", *Trésor-Economics*, No. 182 for a broad survey of the situation. Some research argues that the decline in interest rates is a trend that has been present for several centuries, including P. Schmelzing (2020), "Eight centuries of global real interest rates, R-G, and the 'suprasecular' decline, 1311–2018", Bank of England, *Staff Working Paper*, No. 845.

(8) The European Commission estimated in 2021 that the real euro-area long-term interest rate had fallen by six percentage points over 20 years, whereas potential real GDP growth had fallen by only one percentage point. The aggregate long-term interest rate is the euro area 10-year benchmark government bond yield calculated by the ECB, and the real interest rate is derived using the underlying euro area inflation rate in the over 12 months. See European Commission (2021), "'*r–g*' differentials: latest developments and implications for public debt sustainability", *Fiscal Sustainability Report* 2021, vol. 1.

**Chart 2: Implicit interest rate and nominal growth in France (%)**




*Sources: Jordà-Schularick-Taylor Macrohistory Database, Historical Public Finance Database, WEO Database (April 2023, latest data point 2019), IMF. DG Trésor calculations.*

## 2.3 The interest rate-growth differential could turn positive again in certain advanced countries in 2023 and 2024

The COVID-19 pandemic in 2020 and the energy crisis exacerbated by the war in Ukraine in 2022 have jeopardised the negative *r–g* differential, even though uncertainty about medium-term developments makes it impossible to predict a lasting reversal towards positive *r–g* differentials in the advanced economies. Rising inflation and monetary tightening since 2022 have led to a surge in interest rates at the same time as growth has flagged in the face of various shocks such as worsening terms of trade.[9] The interest rate hikes implemented to fight inflation have an adverse effect on the government's financing terms.[10] France's 10-year sovereign interest rate stood at 3.1% at the end of 2022, compared to 0.2% at the beginning of 2022. This rise in sovereign interest rates affected all advanced economies. Based on the OECD's forecasts for 10-year nominal interest rates in its *Economic Outlook* of June 2023, there is an increasing risk that *r–g* will become positive and that this may happen in 2023 or 2024 for certain advanced economies (e.g. the United States and Italy in 2024 according to the growth assumptions put forward by the IMF in its April 2023 WEO). These real effects are compounded by a nominal effect in the short term. The deterioration of the terms of trade in 2023 has widened the gap between consumer prices and GDP prices, thereby affecting the nominal *r–g* differential. Of course, there is a great deal of uncertainty surrounding these estimated growth rates and 10-year interest rates, which makes it impossible to predict changes in the differential beyond the short term, or even whether it will be negative or positive.

However, it can be affirmed that, by 2027, the rise in sovereign interest rates (the marginal interest cost on debt) will be transmitted to implicit interest rates (the total cost of the debt stock) gradually because of the average maturity of government debt, which stands at 8.5 years in France, after rising steadily from 5.8 years in 2003. The average maturity of government debt in the advanced countries is estimated at 7.3 years.[11] The lengthening of maturities over nearly 20 years can be attributed to market demand for longer target maturities in order to obtain better yields in the prevailing context of very low, and even negative, yields before the pandemic.[12] The transmission of higher sovereign interest rates to the implicit rates is gradual since governments tend to issue long-term securities. This explains why implicit rates are still lower than the 10-year interest rates that governments are currently offering on new bonds. This effect should continue to be felt in 2023 and 2024 and in the medium term. The *r–g* differentials defined using implicit interest rates are very likely to remain negative in the coming years, even though rising interest rates are gradually increasing the risk that the differential may become positive.

(9) See G. Clavères (2022), "The Distribution of Losses Caused by the Energy Terms of Trade Shock", *Trésor-Economics,* No. 318.
(10) Policy rates have a direct effect on the short-term sovereign yields. Long-term sovereign yields reflect expectations about future policy rates, as well as term premiums and inflation expectations.
(11) According to the data in the IMF's Fiscal Monitor (October 2022).
(12) However, the average maturity of France's government debt has not increased when the European Central Bank asset purchasing policies are taken into account. These policies substitute short-term commitments for long-term debt securities.
See P. Copin & J. Dalbard (2022), "France's Sovereign Debt Issuance Strategy", *Trésor-Economics,* No. 297.

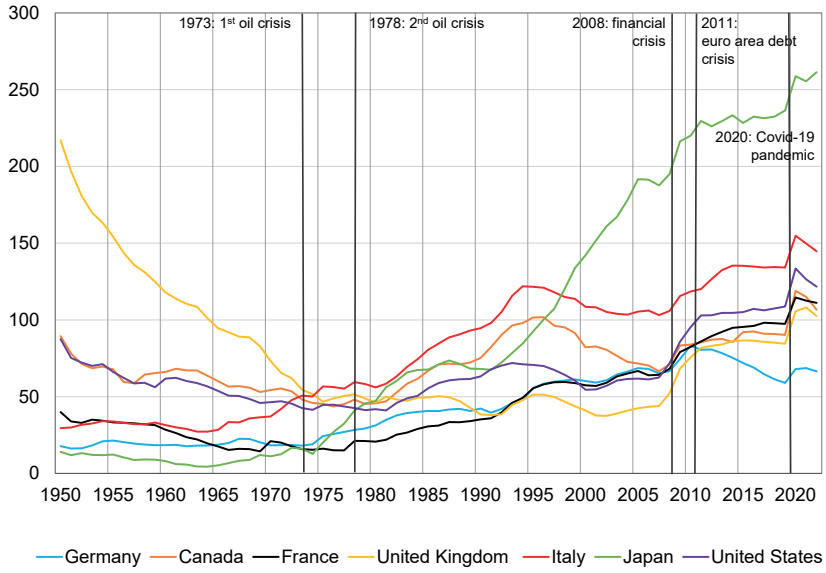
# 3. History shows that *r<g* alone cannot ensure the containment of public debt without a primary surplus

## 3.1 The importance of primary surpluses for reducing public debt

Most advanced economies have come through the pandemic with significantly higher public debt ratios compared to 2019 because of the support measures implemented. An analysis of the conditions that made it possible to reduce the debt ratio after major crises in the past shows that successful debt reduction episodes relied on a combination of negative *r–g* differentials and primary surpluses. In this case, a successful public debt reduction episode is defined as at least three years in a row where the public debt ratio falls by at least 0.5 percentage points of GDP each year. Such episodes occurred mainly in the decades following the Second World War and in the 1990s[13] (see Chart 3).

If we look at the signs of the *r–g* differentials (using the implicit interest rate to identify a negative or positive snowball effect) and the signs of the primary government balances over the a long period (1872- 2019) in each year during such debt reduction episodes), we see that in the vast majority of cases, the primary balance posted a surplus (see Chart 4). In the years with primary surpluses, there was a negative *r–g* differential most of the time; if not, then the primary surplus was large enough to offset the positive snowball effect on debt. In only a small minority of debt reduction episodes was it possible for a negative *r–g* differential to reduce debt despite a primary deficit.

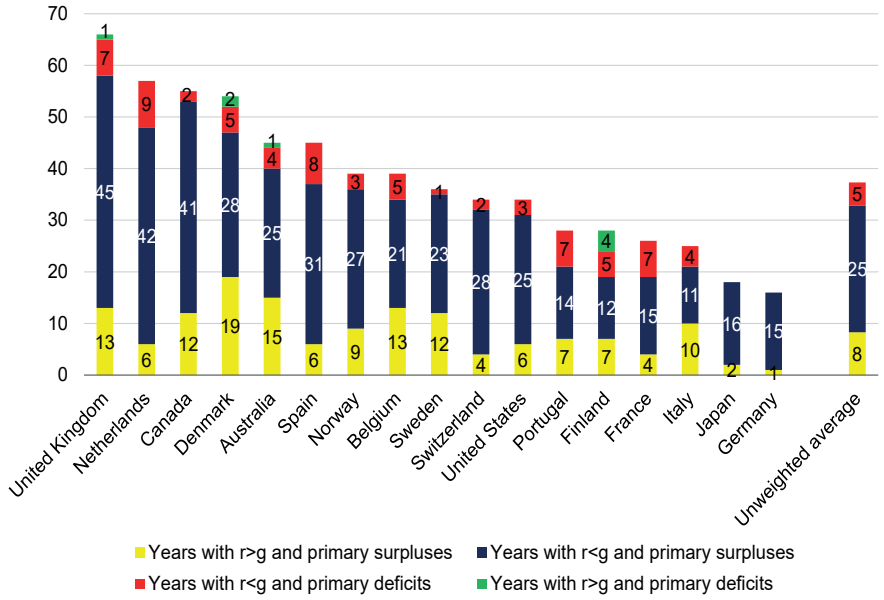
**Chart 3: Government debt stocks (as a percentage of GDP)**




*Sources: Jordà-Schularick-Taylor Macrohistory Database, Historical Public Finance Database, WEO Database (April 2023, latest data point 2021), IMF. DG Trésor calculations. Gross public debt stocks.*

(13) A total of 141 such episodes were identified in a total of 17 advanced countries between 1872 and 2019.

**Chart 4: Number of years included in a debt reduction episode, classified according to *r–g* values and primary balances**




*Sources: Jordà-Schularick-Taylor Macrohistory Database, Historical Public Finance Database, WEO Database (April 2023), IMF. DG Trésor calculations.*

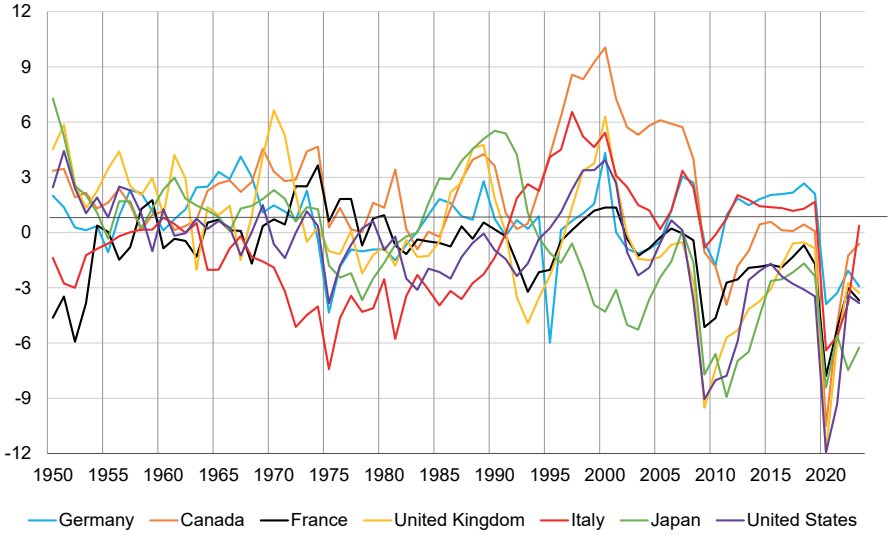
Note: The sample includes G7 countries from 1872 to 2019. A year is counted as part of a debt reduction episode if it is one of at least three years in a row where the public debt ratio fell by at least 0.5 percentage points of GDP per year in a given country. It is possible for a year with a positive *r–g* differential and a primary deficit to be a year included in a debt reduction episode, if a stock-flow adjustment is large enough to lower the ratio.

After massive increases during the Second World War, the G7 countries' public debt ratios posted large declines over three decades to stand at less than 50% of GDP again.[14] The macroeconomic context during the thirty-year post-war boom played a major role in reducing debt ratios with high nominal growth and low interest rates (see Section 2.1). Consequently, the *r–g* differential was very negative and it made the largest contribution to major reductions in public debt ratios, even though large primary surpluses in the advanced economies also played a role that is often overlooked (see Chart 5). Strong and steady economic growth facilitated the primary surpluses. The small number of economic and banking crises meant that governments did not have to introduce recovery or recapitalisation measures that entail major costs for public finances.[15]

In the vast majority of cases, successful fiscal consolidation policies relied on a combination of negative *r–g* differentials and primary surpluses.[16] However, periods with a negative *r–g* differential were not always used to reduce the public debt ratio, for example when the negative snowball effect does not offset the primary deficit, or put differently when the primary balance is smaller than the debt stabilising primary balance (see Box 1). Therefore, a negative *r–g* differential is no guarantee that the public debt ratio will follow a downward trajectory. In the G7 countries between 1950 and 2019, such cases where the negative snowball effect did not offset the primary deficit occurred 13% of the time and in 20% of the years where *r<g*.

(14) The reduction of public debt ratios concerned almost all advanced countries in varying proportions in the years after the Second World War.
(15) See Eichengreen *et al*. (2021), *op. cit.*
(16) This is also the view of the Bank for International Settlements; see BIS (2021), *Annual Economic Report*.

**Chart 5: Primary public balances (as a percentage of GDP)**




*Sources: Jordà-Schularick-Taylor Macrohistory Database, Historical Public Finance Database, WEO Database (April 2023, latest data point 2021), IMF. DG Trésor calculations.*

## 3.2 Future *r–g* values are difficult to predict

The volatility of *r–g* differentials and the possibility of abrupt, or gradual reversals leading to long periods with positive differentials (see Section 2.1), call for a cautious approach to assessing the possibility of a negative differential making a lasting contribution to debt reduction. Furthermore, a reversal of the *r–g* differential is all the more problematical if the debt ratio is high, since the snowball effect is proportional to the debt ratio (see Box 1).

In addition, future changes to the *r–g* differential are all the more uncertain since the differential is dependent on the debt ratio. If the public debt ratio increases, the *r–g* differential increases as well. Mian *et al.* (2021) discuss this "feedback effect" of increased debt on *r–g* in a theoretical framework: when the public debt ratio increases, the convenience yield on public debt, meaning the lower interest rate paid on such debt because of its safety and liquidity, eventually diminishes.[17] The effect of a higher debt ratio on the interest rate-growth differential can be shown empirically. Lian *et al.* (2020) show that a higher public debt ratio leads to shorter negative *r–g* differential episodes and a higher differential on average.[18]

Further analysis on a panel of 18 advanced economies over a long period (1950-2019), while controlling for a number of factors, confirms these results (see Box 2). An increase in the public debt ratio by one percentage point of GDP leads to an average increase of two basis points in the *r–g* differential in the following year. The coefficients found are statistically significant and pass various robustness tests. This order of magnitude is consistent with other research in academic literature, which generally points to an increase of the differential of 1 to 2.5 basis points.[19]

These results also show that, all else being equal, a high debt ratio in a given year reduces the probability that the following year will be a year included in a negative *r–g* episode, such years being defined as one of at least three years in a row where the growth rate is higher than the interest rate.

Therefore, fiscal headroom provided by a favourable snowball effect should not be taken for granted, especially in a context of large increases in public debt since 2019 and rising interest rates.

(17) A.R. Mian, L. Straub & A. Sufi (2022). A goldilocks theory of fiscal deficits (No. w29707). *National Bureau of Economic Research.*
(18) W. Lian, W. Presbitero and U. Wiriadinata (2020), "Public Debt and *r–g* at Risk", IMF *Working Paper*, No. 137.
(19) See the review by Mian *et al.* (2021), *op. cit.*

## Box 2: Econometric study of the link between *r–g* and the debt ratio

We use a methodology inspired by Mian *et al.* (2021) and Lian *et al.* (2020) to assess the effect of an increase in the public debt ratio on the interest rate-growth differential.

*Data*

The panel includes 18 advanced economies: Australia, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, United States. The *r–g* differential is the difference between the nominal 10-year interest rate of country *i* and the nominal GDP growth rate in year *t*.[a] The data used for the interest rate *r* is taken from the Jordà-Schularick-Taylor long-run macroeconomic database for 1950-2011, and from the OECD database for 2012-2019.

The Jordà-Schularick-Taylor database is used for the nominal growth rate from 1950 to 2011 and the IMF's WEO database (April 2023) is used for the nominal growth rate from 2012 to 2019. Not all of the data is available for all of the years and for all of the countries (non-balanced panel). The nominal growth rate is also used to construct the dummy variable $D1_{i,t}$, used to account for years when the nominal growth rate abruptly fell. The value of this dummy variable is 1 if the growth rate is in the lowest decile for the country for the whole period and 0 otherwise. For the whole panel, the value of the dummy variable is 1 for approximately 10% of the sample. In order to exclude extreme values for *r–g*, during wars, for example, we eliminate years where the absolute value of *r–g* exceeds the average by country for the whole period by more than four standard deviations (accordingly six observations were eliminated from the total sample).

The public debt ratio as a percentage of GDP is taken from the Historical Public Finance Database, the IMF database that covers public finance from 1950 to 2011.[b] The IMF's WEO data is used for the period from 2012 to 2019.

The public debt ratio as a percentage of GDP is taken from the Historical Public Finance Database, the IMF database that covers public finance from 1950 to 2011. The IMF's WEO data is used for the period from 2012 to 2019.

### 1. Effect of the public debt ratio on *r–g*

*Specifications*

The explained variable is the *r–g* differential in the current year and the explanatory variables are the public debt ratio as a percentage of GDP, lagged by one year, along with the dummy variable for recessions and fixed effects. By taking the previous year's debt ratio, we prevent the interest rate and growth rate of the current year (the explained variable) from affecting the debt ratio as a percentage of GDP (the explanatory variable) in the same year. The regressions are run on the panel with and without fixed effects, using an ordinary least squares method with different specifications for the fixed effects.

*Results*

Generally speaking, the results show that the level of the public debt ratio has a significant effect on *r–g*: the coefficients are positive and statistically significant under all specifications (see Table 3). Under the specification with fixed time and country effects, the coefficient shows that, on average and with the other variables constant, for an increase of one percentage point in the debt ratio, *r–g* increases by two basis points in the following year.

The results are robust to the exclusion of the dummy variable for recessions and to a change in period (1872-2019) or a change in the countries included in the panel. If the G7 countries are considered on their own, the coefficients are of the same order of magnitude, but they are weaker and less significant.

a. The 10-year interest rate is used rather than the implicit or effective interest rate, because the 10-year rate responds more rapidly to changes in financing terms, as pointed out by Lian *et al*. (2002).
b. See https://www.macrohistory.net/database/

**Tableau 3 : Résultats**

| $(r - g)_{i,t}$ | (1) | (2) | (3) |
|---|---|---|---|
| $\left(\frac{debt}{GDP}\right)_{i,t-1}$ | **0.028*****<br>(0.010) | **0.037*****<br>(0.010) | **0.02*****<br>(0.005) |
| $D1_{i,t}$ | 0.1***<br>(0.008) | 0.1***<br>(0.008) | 0.09***<br>(0.01) |
| Observations | 1253 | 1253 | 1253 |
| $R^2$ adjusted | 0.12 | 0.24 | 0.55 |
| Fixed effects | | Country | Country and time |

****p<0.001; **p<0.01; *p<0.05; standard deviations are robust to heteroskedasticity and to autocorrelation (Newey West method). Standard errors in parentheses.*

## 2. Effect of the debt ratio on the probability of negative *r–g* episodes

Negative *r–g* episodes are defined as at least three years in a row where *r*<*g*. For the 18 countries in the panel over the period from 1950 to 2019, there were 82 episodes where *r*<*g*, accounting for approximately 27% of the time, and the average *r*<*g* episode lasted for eight years. A probit model with three specifications with different fixed effects is used to assess the effect that the debt ratio, with the other variables constant, has on the probability that the following year will be part of an *r*<*g* episode.

The results suggest that a public debt ratio that is higher than average for the country significantly reduces the probability that the following year will be part of an *r*<*g* episode, with the other variables remaining constant.[c] The coefficients related to the debt ratio are negative and significant. They are robust to the inclusion of fixed effects (see Table 4).

**Table 4: Probability of a negative *r–g* episode**

8 countries, 1950-2019

| $\mathbb{N}_{i,t}$ | (1) | (2) | (3) |
|---|---|---|---|
| $\left(\frac{debt}{GDP}\right)_{i,t-1}$ | **–0.006*****<br>(0.001) | **–0.008*****<br>(0.002) | **–0.007****<br>(0.002) |
| $D1_{i,t}$ | –5.38***<br>(0.06) | –5.48***<br>(0.09) | –5.74***<br>(0.28) |
| Observations | 1253 | 1253 | 1253 |
| Fixed effects | | Country | Country and time |

****p<0.001; **p<0.01; *p<0,05, standard deviations are robust to heteroskedasticity.*
How to read this table: The coefficients obtained with the probit cannot be interpreted directly, but only in relation to a benchmark situation, because of the specificity of the model.

The higher the debt ratio is one year; the smaller the probability that the following year will be part of an *r*<*g* episode. For example, the estimates for the sample used under specification (1) show that when the current year's debt ratio is 50% of GDP, the probability that the following year will be part of an *r*<*g* episode is 36%, versus 26% when the debt ratio is 100%. These estimates call for caution and are used to illustrate a causality direction, since the model cannot capture the effects on the coefficients of the other explanatory variables that are not included, nor does it capture the effect of the time elapsed since the beginning of an ongoing episode on the probability that it will end in the following year.

**Publisher:**
Ministère de l'Économie,
des Finances
et de la Souveraineté
industrielle et numérique
Direction générale du Trésor
139, rue de Bercy
75575 Paris CEDEX 12

**Publication manager:**
Dorothée Rouzet

**Editor in chief:**
Jean-Luc Schneider
(01 44 87 18 51)
tresor-eco@dgtresor.gouv.fr

**English translation:**
Centre de traduction
des ministères économique
et financier

**Layout:**
Mimose Mellia
ISSN 1962-400X
eISSN 2417-9698

## Recent Issues in English

**September 2023**

N° 332 World Economic Outlook in Autumn 2023: The Economy is Holding out Against Rising Interest Rates

Raphaël Beaujeu, Louis Bertrand, Adama Hawa Diallo, Juliette Flament, Paul Mainguet, Cyprien Ronze-Spilliaert, Eloïse Villani

**August 2023**

N° 331 The Safe Assets Market

Agathe Blanchard, Clara Léonard, Léonore Raguideau-Hannotin

**https://www.tresor.economie.gouv.fr/Articles/tags/Tresor-Eco**

**Direction générale du Trésor**

**@DGTresor**

**To subscribe to *Trésor-Economics*: bit.ly/Trésor-Economics**