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# "Financial Intermediation, Capital Accumulation and Crisis Recovery"

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# Financial Intermediation, Capital Accumulation and Crisis Recovery*

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**Abstract**

This paper integrates banks into a two-sector neoclassical growth model to account for the fact that a fraction of firms relies on banks to finance their investments. There are four major contributions to the literature: First, although banks' leverage amplifies shocks, the endogenous response of leverage to shocks is an automatic stabilizer that improves the resilience of the economy. In particular, financial and labor market institutions are essential factors that determine the strength of this automatic stabilization. Second, there is a mix of publicly financed bank re-capitalization, dividend payout restrictions, and consumption taxes that stimulates a Pareto-improving rapid build-up of bank equity and accelerates economic recovery after a slump in the banking sector. Third, the model replicates typical patterns of financing over the business cycle: procyclical bank leverage, procyclical bank lending, and countercyclical bond financing. Fourth, the framework preserves its analytical tractability wherefore it can serve as a macro-banking module that can be easily integrated into more complex economic environments.

**JEL:** E21, E32, F44, G21, G28

**Keywords:** Financial intermediation, capital accumulation, banking crisis, macroeconomic shocks, business cycles, bust-boom cycles, managing recoveries.

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# 1 Introduction

Financial frictions affect the propagation of economic shocks and are an essential factor for understanding short-run dynamics and long-run macroeconomic performance. Typically, financial frictions can be traced back to either contract enforceability problems or asymmetric information and – on this ground – give rise to levered finance to align the interest of borrowers and lenders.[1]

Since the seminal contributions of Bernanke and Gertler (1989), Bernanke et al. (1996) and Kiyotaki and Moore (1997), it is well-understood that in an economy with financial frictions, even small temporary shocks can have large and persistent effects on economic activity by impacting the net worth of levered agents. In this literature, firms need net worth to credibly commit to the contractual obligations of the credit contract. Deteriorating conditions reduce firm profits, net worth and, thus, the capacity to obtain credit. The propagation of shocks through net worth and firm credit may have large and persistent impact on economic activity – a mechanism referred to as the *credit channel*.[2]

Although Holmström and Tirole (1997) extended the analysis to financial intermediaries, it was not until the 2007-2009 financial and banking crisis that macroeconomists took up their proposal. Financial intermediaries channel funds from investors to entrepreneurs, cope with the underlying financial friction and are, at the same time, subject to frictions themselves. Banks have to hold equity capital to credibly commit to the contractual obligations of the deposit contract. Specifically, the level of bank equity is the *skin in the game* which determines the capacity to attract loanable funds. When financial conditions deteriorate, bank profits decline, which negatively affects future bank equity holdings and, thus, the future capacity to attract loanable funds and to supply loans to entrepreneurs. The propagation of shocks through the bank balance sheets has real large and persistent impact on economic activity – a mechanism referred to as the *bank lending channel*.[3] In essence, the *bank lending channel* is a propagation mechanism similar to the *credit channel*, but it impacts different borrowers.[4]

In this paper, we develop a two-sector neoclassical growth model with financial frictions in the tradition of Holmström and Tirole (1997). The model has microfounded levered

[1] See Quadrini (2011) for an excellent overview of the extensive literature on financial frictions and macroeconomic performance.

[2] This literature includes Carlstrom and Fuerst (1997) and, more recently, Cooley et al. (2004), Christiano et al. (2007), Jermann and Quadrini (2012), Brumm et al. (2015), and Gomes et al. (2016).

[3] This literature includes Van den Heuvel (2008), Meh and Moran (2010), Gertler and Kiyotaki (2010), Gertler and Karadi (2011), Rampini and Viswanathan (2017), Brunnermeier and Sannikov (2014), and Quadrini (2014).

[4] There are also notable deviations from this approach to model banking systems in macroeconomic context, see e.g. Angeloni and Faia (2013) and Acemoglu et al. (2015).

banks and allows for two forms of finance – bonds and loans. We adopt a medium- to long-run perspective in the sense that output reacts smoothly to adverse shocks and economic dynamics are essentially driven by capital re-allocation and accumulation instead of abrupt changes in prices. We contribute to the literature in four respects.

First, we provide novel insights into the *bank lending channel*. We show that although the level of leverage is an amplification mechanism of shocks, the endogenous response of leverage to productivity and capital shocks is an automatic stabilizer that improves the resilience of the economy to adverse shocks. Specifically, suppose there is a shock that – directly or indirectly – leads to a decline in bank equity. Investors, *ceteris paribus*, reduce their deposits to restore the initial bank leverage, i.e. loan supply decreases. As a result, capital productivity in the loan financed sector increases and so do bank profits. The effective financial friction loosens such that investors can increase their deposits without incentivizing banks to defect. The ensuing increase in bank leverage partially neutralizes the initial decline in loan supply.

In particular, we show that financial market institutions (e.g. capital requirements) and labor market institutions (e.g. labor mobility and employment protection legislation) affect the elasticity of bank leverage with respect to productivity and capital shocks and, therefore, the resilience of the financial system. While the impact of financial market institutions on labor markets is well-understood, we show that there is a non-negligible feedback effect from labor market institutions to credit market conditions and the resilience of the financial system – a result unique to the macro-banking and macro-labor literature.

Second, we derive macro-prudential policies comprising investor-financed re-capitalization of banks, dividend payout restrictions, consumption taxes, and investment subsidies that are Pareto-improving and speed up the economic recovery after a banking crisis, without encouraging banks to take excessive leverage in the expectations of future bailouts. In a similar vein, Acharya et al. (2017) show that bank equity capital has the characteristics of a public good which justifies dividend pay-out restrictions to internalize the impact of dividend payments on social welfare and output. In fact, bank-recapitalization and dividend pay-out restrictions have been used during the 2007 – 2009 financial and banking crisis in the United States and, as Shin (2016) points out, during the 2007 – 2014 financial and banking crisis in Europe.

Third, the model replicates typical patterns of financing over the business cycle: procyclical bank leverage, procyclical bank lending and countercyclical bond financing – see Adrian and Shin (2014), Adrian and Boyarchenko (2012), Adrian and Boyarchenko (2013) and Nuño and Thomas (2012) for empirical evidence. This holds if downturns are associated with negative productivity, bank equity or trust shocks – or any combination thereof. Moreover, when recessions are accompanied by a sharp temporary decline in bank equity capital, they are deeper and more persistent than regular recessions – a result that is consistent with the findings in Bordo et al. (2001), Allen and Gale (2009), and Schularick and Taylor (2012).

Fourth, our model provides an analytically tractable macro-banking module that can easily be integrated into more complex economic environments to give more account for the special roles of banks in macroeconomic analysis.

Financial frictions are at the core of our macro-banking model: they provide a microfoundation for the existence of banks and play an essential role for the propagation of adverse shocks.[5] Specifically, there are two production sectors. Firms in sector $I$ (*intermediary financed*) are subject to severe financial frictions, which prevents them from obtaining financing directly through the financial market. As banks alleviate the moral hazard problems resulting from these financial frictions, firms in sector $I$ obtain bank loans instead. However, bank lending itself is limited, as bankers can only pledge a fraction of their revenues to depositors and are thus subject to a different financial friction that gives rise to an endogenous leverage constraint which depends on equilibrium capital returns in sector $I$ and the deposit rate. Firms in sector $M$ (*market financed*) are not subject to financial frictions and issue corporate bonds. The need for bank lending – also called *informed lending* – coupled with the lack of full revenue pledgeability, are the two financial frictions in our model.[6] In the baseline model, there are three types of agents: investors, bankers and workers. The latter are immobile across production sectors as their skills are sector-specific. Workers do not save and consume their entire labor income. Investors and bankers have standard intertemporal preferences and decide in each period how much to save and to consume.[7] Their utility maximization problems yield two accumulation rules for investor wealth and bank equity, respectively. These rules are coupled in the sense that the investor's saving and investment policies depend on how bankers fare and vice versa. Both types of lending – *informed lending* by banks and *uninformed lending* through capital markets – enable capital accumulation in the respective sectors.

The paper is organized as follows. Section 2 relates our paper to the existing literature.

[5]Gersbach and Rochet (2017) study a static version of the same banking model in which bank equity capital cannot be accumulated. Gersbach et al. (2016) integrate banks into the Solow growth model.

[6]As we discuss in Section 3.3, the foundation of these frictions can be moral hazard problems à la Holmström and Tirole (1997), asset diversion (as in Gertler and Karadi (2011) and Gertler and Kiyotaki (2010)) or non-alienability of human capital (as in Hart and Moore (1994) and Diamond and Rajan (2000)).

[7]In the extensions, we consider a version of the model in which there are only two types of agents: households, acting as investors and workers, and banks. To preserve clarity in exposition, we solely use the terminus *household* for that case.

Section 3 introduces the model, Section 4 defines and characterizes sequential market equilibria, and Section 5 analyzes the steady state allocation. Section 6 establishes global stability, characterizes global economic dynamics, and analyzes the propagation of adverse shocks when bank leverage reacts sensitive to equilibrium conditions. Section 7 derives public policies and financial regulation to speed up recoveries when the economy is hit by a negative shock to bank equity capital. Section 8 provides a quantitative analysis to illustrate the static and dynamic effects that have been derived in the previous sections of this paper. Section 9 summarizes and concludes. Several extensions to the model are relegated to Appendix D.

## 2 Relation to the Literature

Our paper is closely related to three recent strands of the literature that integrate financial intermediation into macroeconomic models. The objective is to analyze the propagation of shocks through bank balance sheets and to derive policies to manage financial and banking crises.

First, our paper is most closely related to recent research that integrates financial intermediation into the neoclassical growth model, e.g. Van den Heuvel (2008), Gertler and Kiyotaki (2010), Rampini and Viswanathan (2017), Brunnermeier and Sannikov (2014), Quadrini (2014), and Acemoglu et al. (2015). Brunnermeier and Sannikov (2014) have stressed that the economy's reaction to adverse shocks can be highly non-linear. Specifically, if the economy is sufficiently far away from its steady state, even small shocks can generate substantial amplification and endogenous fluctuations. In contrast, near the steady state, the economy is resilient to most shocks. He and Krishnamurthy (2013) find similar non-linear effects when risk premia on equity increase sharply as financial constraints become binding. Rampini and Viswanathan (2017) develop a dynamic theory of financial intermediaries acting as collateralization specialists, in which credit crunches are persistent and can delay or stall economic recoveries. They consider a one-sector economy with risk-neutral agents and show that – under certain conditions – there are large reactions to small changes in interest rates. In contrast to Rampini and Viswanathan (2017), we develop a two-sector neoclassical growth model with levered financial intermediation where savings, investments, interest rates, and bank capital accumulation react more smoothly to shocks for three reasons:

First, with an alternative investment opportunity that does not rely on levered finance, investors re-optimize their portfolio, thereby attenuating the immediate impact of an adverse shock. Second, as investors are risk-averse, they smooth consumption and spread the immediate shock to several periods. Third, leverage itself reacts endogenously and immediately accommodates the banks' lending capacity to smooth out adverse shocks to the bank balance sheet. Nevertheless, the special role of banks in the capital accumulation process with binding leverage constraints as well as the potentially divergent reactions of investor wealth and bank equity capital generate sizeable and persistent output reactions, as bank profits and thus future lending capacities are affected. In this sense, our approach adopts a medium- to long-run perspective on how economies with a large bank-financed sector react to shocks, because economic dynamics are driven by adjustments in capital accumulation instead of abrupt changes in price levels. Another difference with Rampini and Viswanathan (2017) is that in our model, the relative capital productivity of financially constrained and unconstrained firms is endogenously determined by the joint evolution of bank equity and investor capital. The mix of bond and loan finance evolves endogenously and replicates typical financing patterns over the business cycle: counter-cyclical bond-to-loan finance ratios (see De Fiore and Uhlig (2011)) and pro-cyclical bank leverage (see Adrian and Shin (2014)). In addition, an increase of the financial frictions induces a recession in the economy, but leads to a boom in the banking sector that can – under certain conditions – even trigger a boom in the economy in the medium-run.

Second, our paper is closely related to a recent strand in the literature that integrates banks into New-Keynesian DSGE models, e.g. Meh and Moran (2010), Gertler and Karadi (2011), and Angeloni and Faia (2013). Meh and Moran (2010) and Angeloni and Faia (2013) have provided valuable insights about the bank capital transmission channel. Meh and Moran (2010) find that this channel amplifies the impact of technology shocks on inflation and output, and delays economic recovery. Angeloni and Faia (2013) introduce a fragile banking system, in which banks are subject to runs, into a new-Keynesian DSGE model. They show that a combination of counter-cyclical capital requirements and monetary policies responding to asset prices or bank leverage is optimal in the sense that it maximizes the *ex-ante* expected value of total payments to depositors and bank capitalists. In contrast to this strand of literature, we abstract from price rigidities and develop a parsimonious neoclassical macro-banking model that exhibits smooth reactions to adverse shocks. In contrast to Angeloni and Faia (2013), we focus on incentive compatible *ex-post* policies to manage financial and banking crises instead of *ex-ante* policies to prevent them.

Third, in terms of policy implications, our paper is closely related to Martinez-Miera and Suarez (2012), who study a dynamic general equilibrium model in which banks decide *inter alia* on their exposure to systemic shocks. Capital requirements reduce the direct impact of negative systemic shocks, but they also lower credit supply and output in normal times: optimal capital requirements balance these costs and benefits. Our model is complementary to Martinez-Miera and Suarez (2012) and considers the simultaneous build-up of bank equity and investor wealth after both, anticipated and unanticipated shocks to productivity, wealth, and financial frictions. In contrast to Martinez-Miera and Suarez (2012), who focus on capital requirements and crisis prevention, we focus on crisis management and show that a revenue-neutral combination of investor-financed bank re-capitalization, publicly enforced dividend payout restrictions, consumption taxes and saving subsidies can speed up the recovery after a banking crisis and, in particular, can make workers and investors better off, while leaving the welfare of bankers unaffected. In a similar vein, Itskhoki and Moll (2014) study how taxes or subsidies may favorably impact the transition dynamics in a standard growth model with financial frictions. Our study is complementary, as we focus on two policies that are typically applied in banking crises: re-capitalization of banks and dividend payout restrictions. Acharya et al. (2011) study dividend payments of banks in the 2007 – 2009 financial crisis and argue that early suspension of dividend payments can prevent the erosion of bank capital in the future. In a similar vein, Acharya et al. (2017) and Onali (2014) suggest that because dividend payments exert externalities on other banks, dividend payout restrictions can adjust for the negative external effect.

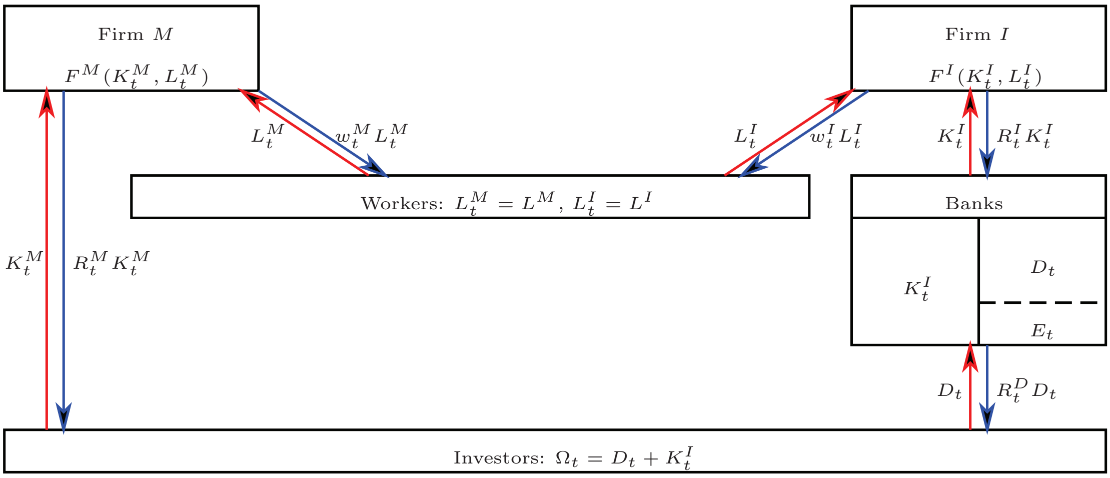
# 3 Model

We integrate a simple model of banks into a two-sector neoclassical growth model. Time is discrete and denoted by $t \in \{0, 1, 2, \dots\}$. There are two production sectors, with constant returns to scale technologies using capital and labor to produce a homogenous good that can be consumed or invested. Sectors differ with respect to their access to capital markets: while firms in sector $M$ (*market-financed* or *bond-financed*) can borrow frictionlessly on the capital market, firms in sector $I$ (*intermediary-financed* or *loan-financed*) have no direct access to financial markets and rely on bank loans instead. Banks monitor entrepreneurs in sector $I$ and enforce the contractual obligation from the loan contract. Banks themselves are subject to financial frictions that limit the amount of loanable funds these banks can attract. Consumption is the numéraire: its price is normalized to 1. There are three types of agents: workers, investors, and bankers.[8] Workers are hand-to-mouth consumers who consume their entire labor income instantaneously. In contrast, investors and bankers choose consumption and investment

[8]Splitting the household sector into workers and investors preserves the analytical tractability of the model. We also consider a variation of the model in Section 9 and Appendix D in which there is only one type of households that supplies labor and acts as investor. We show that this variation leaves the steady state allocation unaffected and yields model dynamics that are qualitatively and quantitatively at a similar order of magnitude.

to maximize lifetime utility. The general structure of the model is depicted in Figure 1 and the details are set out in this section.

Figure 1: General Structure of the Model




## 3.1 Production

Production takes place in two different sectors labeled as sector $M$ and sector $I$. Both sectors consist of a continuum of identical firms. The production technologies exhibit constant returns to scale in the production factors capital and labor, have positive and diminishing marginal returns regarding a single production factor and satisfy the Inada conditions. Because of constant returns to scale and competitive markets, we focus on a price-taking representative producer in each sector, without loss of generality. Specifically, the aggregate production technologies are Cobb-Douglas and given by

$$Y_t^j = z^j A_t (K_t^j)^\alpha (L_t^j)^{1-\alpha}, \quad j \in \{M, I\},$$

where $A_t$ is an index of the economy-wide common total factor productivity, $z^j$ is an index of sectoral total factor productivity, $\alpha$ $(0 < \alpha < 1)$ is the output elasticity of capital, and $K_t^j$ and $L_t^j$ denote capital and labor input in sector $j \in \{M, I\}$, respectively.

Firms in sector $M$ can borrow frictionlessly on capital markets by issuing corporate bonds. Firms in sector $I$ have neither the reputation nor the transparency to resolve information asymmetries, such that they cannot credibly pledge repayment to investors. A severe moral hazard problem between investors and firms in sector $I$ ensues, which leads to the exclusion of the latter from capital markets. These firms, however, can obtain loans from financial intermediaries that monitor them and enforce the contractual obligation.[9] For simplicity, we assume that banks can ensure full repayments of bank loans.[10]

Taking interest and wage rates as given, the representative firm in each sector $j \in \{M, I\}$ chooses capital and labor to maximize its period profit

$$\max_{\{K_t^j, L_t^j\}} \left\{ z^j A_t (K_t^j)^\alpha \ (L_t^j)^{1-\alpha} - r_t^j K_t^j - w_t^j L_t^j \right\}, \quad j \in \{M, I\}, \tag{1}$$

where $w_t^j$ is the wage rate and $r_t^j$ is the rental rate of capital in sector $j$ and period $t$, respectively. We further define $K_t \doteq K_t^M + K_t^I$ and $L_t \doteq L_t^M + L_t^I$ as total capital and total labor used in production.

### 3.2 Workers and Investors

There is a continuum of workers with mass $L$ ($L > 0$). Each worker is endowed with one unity of labor, of which he inelastically supplies $l^M$ and $l^I = 1 - l^M$ units to firms in sectors $M$ and $I$, respectively. Workers are hand-to-mouth consumers, i.e. they consume their entire labor income and do not save.[11] We focus on a representative worker who takes wages as given and earns $w_t^M L^M + w_t^I L^I$, where $L^M = l^M L$ and $L^I = l^I L = L - L^M$.

The assumption of sector-specific inelastic labor supply can be understood in several ways: First, as a lack of transferability of skills across sectors and, second, as a manifestation of imperfect labor markets which itself may be caused by lack of mobility of workers. There exists a large recent literature on sector- or task specific skill and their impact on structural change, wages and employment.[12] As a consequence, wage differentials between both sectors can be substantial and persistent, and are driven by the joint accumulation of bank equity capital and investor wealth. The labor market imperfection in combination with the Inada conditions ensures that there will be no concentration in either of the two production sectors in the long-run, even when sector-specific productivities $z^j$ differ.[13]

[9] Firms that rely on bank credit (sector $I$) are typically younger and smaller than the firms in sector $M$ (see e.g. De Fiore and Uhlig (2015)).

[10] Limited pledgeability of loan repayments by firms in sector $I$ can easily be incorporated by adding the non-pledgeability part to the financial friction we discuss in subsection 3.3.

[11] There are several reasons for which workers may not want to save and behave like hand-to-mouth consumers, e.g. lower discount factors or borrowing constraints. For the purposes of our analysis, we do not need to assess the specific reason. As reported in Challe and Ragot (2016), estimates of the share of hand-to-mouth households in the United States vary a lot and range from 15% to 60%. A recent study by Kaplan et al. (2014) finds that more than one-third of the population in the Unites States decides to save little or nothing.

[12] See, e.g. Acemoglu and Autor (2011) and Bárány and Siegel (2017).

[13] Alternatively, a more complex production system, with sectors $M$ and $I$ producing two distinct

There is a continuum of investors with unit mass. Each investor is endowed with some units of the capital good which can be used for investment in bonds and deposits and for consumption. In the absence of labor income, disposable income is linear homogenous in wealth, and because the period-utility is logarithmic, consumption and saving decisions are linear homogenous in wealth, too. This implies that the distribution of capital among investors has no impact on aggregate consumption, saving, and investment, such that we can restrict the analysis to a representative investor without loss of generality.[14] At the beginning of period 0, the representative investor is endowed with $\Omega_0$ units of capital. He chooses a sequence of investment into bonds and deposits $\{B_t, D_t\}_{t=0}^{\infty}$ at the beginning of a period, consumption $\{C_t^H\}_{t=0}^{\infty}$, and savings $\{\Omega_{t+1}\}_{t=0}^{\infty}$ at the end of a period to maximizes his lifetime utility subject to the sequential budget constraint. The utility maximization problem is given by

$$\max_{\{C_t^H, \Omega_{t+1}, D_t, B_t\}_{t=0}^{\infty}} \left\{ \sum_{t=0}^{\infty} \beta_H^t \ln(C_t^H) \right\} \tag{2}$$

subject to

$$\begin{aligned} & C_t^H + \Omega_{t+1} = r_t^M B_t + r_t^D D_t + (1-\delta)\Omega_t \\ & B_t + D_t = \Omega_t \\ & \Omega_0 \text{ given,} \end{aligned}$$

where $r_t^M$ and $r_t^D$ denote the return to bonds and deposits, respectively, $\delta$ is the capital depreciation rate, and $\beta_H = \frac{1}{1+\rho_H}$ $(0 < \beta_H < 1)$ denotes the discount factor and $\rho_H$ the discount rate.

Due to the Inada condition and imperfect labor markets, any equilibrium allocation must have strictly positive capital in both production sectors. As a result, in the absence of risk, investors must be indifferent between deposits and bonds which implies $r_t^D = r_t^M$, such that the representative investor's budget constraint simplifies to

$$C_t^H + \Omega_{t+1} = \Omega_t(1 + r_t^M - \delta). \tag{3}$$

intermediate goods that are complementary production factors for a final good sector would serve the same purpose. In that case, the additional market imperfection would be on the market for intermediate goods instead of the labor market.

[14]See Alvarez and Stokey (1998), Krebs (2003a), and Krebs (2003b) for a general derivation of this result.

### 3.3 Bankers

There is a continuum of bankers and each banker owns and runs a financial intermediary. Bankers can alleviate the moral hazard problem of the entrepreneurs in sector $I$, as they evaluate and monitor entrepreneurs and enforce contractual obligations. The costs of these activities are neglected.[15] Bankers themselves raise funds from investors at the deposit rate but cannot pledge the entire amount of repayments from entrepreneurs to investors, i.e. bankers are subject to a moral hazard problem themselves. Specifically, if the banker has granted a loan of size $k_t^I$ to entrepreneurs, we assume that $\theta k_t^I$ of the revenues are non-pledgeable. In essence, parameter $\theta \in (0,1)$ provides a concise measure of the financial friction between bankers and depositors.[16]

At the beginning of period $t$, a typical banker owns $e_t$ which she uses as equity funding for her bank. She attracts additional funds $d_t = k_t^I - e_t$ from investors and lends $k_t^I$ to entrepreneurs in sector $I$.[17] Note that equity $e_t$ is inside equity only, i.e. banks cannot raise equity on the market to improve their lending capacity. This assumption simplifies our analysis without interfering with our main insights, as we mainly focus on financial and banking crises, i.e. times in which banks are under distress, and raising new equity is expensive on the ground of a standard pegging order argument.[18]

In order to attract loanable funds $k_t^I - e_t$ from investors, a banker has to be able to pledge at least $(1 + r_t^M)(k_t^I - e_t)$ to investors, as they would otherwise solely invest into bonds. Because $\theta k_t^I$ of revenues is non-pledgeable, incentive compatibility of the deposit contract requires that the banks profit from fulfilling the contractual obligation exceeds

[15] We discuss the impact of intermediation cost on the steady state allocation in Appendix D and show that while bank leverage and return on equity are unaffected by intermediation cost, this cost nevertheless reduces steady state investor wealth, bank equity capital and production.

[16] The partial non-pledgeability of revenues leads to moral hazard between bankers and investors as in Holmström and Tirole (1997) and can alternatively be traced back to the possibility of asset diversion (as in Gertler and Karadi (2011) and Gertler and Kiyotaki (2010)) or non-alienability of human capital (as in Hart and Moore (1994) and Diamond and Rajan (2000)). See Gersbach and Rochet (2013) for an extensive discussion of different mechanisms that micro-found moral hazard in the banker-depositor relationship. Furthermore, assume that when bankers shirk in the current period, they cannot be excluded from seeking new funds from investors in the next period. This rules out that bankers can pledge revenues from future periods in order to attract more funds today. For example, consider the case of asset diversion. Suppose that a banker attempts to pledge $(1-\theta')k_t^B$ in the current period with $\theta' < \theta$ in a long-term contract with more than one period, in which she invests $k_t^B$ more than once. Then, she can divert $\theta k_t^B$ in period $t$ and seeks new funds in period $t+1$. This is profitable and thus $(1-\theta')k_t^B$ cannot be pledged.

[17] In principle, bankers could also invest their resources in sector $M$. However, this will not occur when the leverage constraint binds, as bank equity is scarce in such circumstances and sector $I$ pays higher returns on investments to bankers.

[18] Our approach is common in the literature, which often follows a similar line of argument, e.g. Meh and Moran (2010) or Gertler and Kiyotaki (2010). A notable extension is Ellingsen and Kristiansen (2011) who develop a static banking model with inside equity, outside equity and deposits.

the benefit from retaining the non-pledgeable part of the investment. Thus,

$$(1 + r_t^I)k_t^I - (1 + r_t^M)(k_t^I - e_t) \geq \theta k_t^I,$$

which can be rewritten as

$$k_t^I \leq \frac{1 + r_t^M}{r_t^M - r_t^I + \theta} e_t. \tag{4}$$

Condition (4) is the market imposed leverage constraint and follows from the investors' decision to limit the supply of loanable funds in order to incentivize the banker to comply with the contractual obligations of the deposit contract.

Suppose that total bank equity $E_t$ is relatively scarce. In this case, loan supply is limited by low bank equity capital, which leads to under-investment in sector $I$. Therefore, leverage constraints are binding and $r_t^I > r_t^M$. In this situation, a banker is always better off attracting loanable funds and investing $k_t^I = e_t + d_t$ in sector $I$, thereby earning $(1 + r_t^I)(e_t + d_t) - (1 + r_t^M)d_t$, than, first, investing only $e_t$ in sector $I$ thereby earning $(1 + r_t^I)e_t$ or, second, investing in sector $M$ thereby earning $(1 + r_t^M)(e_t + d_t) - (1 + r_t^M)d_t$. Because individual bankers are price takers, profit maximization implies that bankers lever as much as possible and Condition (4) holds with equality. Note that the binding leverage constraint is linear in bank equity and aggregation is straightforward. Therefore, without loss of generality, we focus on a price taking representative banker facing an aggregate leverage constraint

$$K_t^I = \frac{1 + r_t^M}{r_t^M - r_t^I + \theta} E_t. \tag{5}$$

Defining bank leverage as

$$\lambda_t \doteq \frac{1 + r_t^M}{r_t^M - r_t^I + \theta}, \tag{6}$$

we rewrite condition (5) more compactly as $K_t^I = \lambda_t E_t$. We will establish the formal condition on scarcity of bank equity in Section 4.1. At the current stage, we simply define $\Gamma \subseteq \mathbb{R}_+^2$ as the partition of the state space $(E_t, \Omega_t)$ for which leverage constraints are binding.

Alternatively, suppose that total bank equity $E_t$ is relatively abundant, such that leverage constraints are non-binding, i.e. $(E_t, \Omega_t) \in \mathbb{R}_+^2 \setminus \Gamma$. In this case, loan supply is not limited by the level of bank equity, such that competitive capital markets push down the returns in sector $I$ until interest rates in both sectors get aligned: $r_t^I = r_t^M$.

The bank's disposable income at the end of the period is $\theta K_t^I - \delta E_t = (\theta\lambda_t - \delta)E_t$ when leverage constraints are binding and $(1 + r_t^M - \delta)E_t$ when leverage constraints are non-binding. The representative banker has logarithmic period-utility. Given her initial endowment $E_0$, she chooses a sequence of consumption $\{C_t^B\}_{t=0}^{\infty}$ and savings $\{E_{t+1}\}_{t=0}^{\infty}$ to maximize her lifetime utility. The utility maximization problem is given by

$$\max_{\{C_t^B, E_{t+1}\}_{t=0}^{\infty}} \left\{ \sum_{t=0}^{\infty} \beta_B^t \ln(C_t^B) \right\} \tag{7}$$

subject to

$$C_t^B + E_{t+1} = \begin{cases} (\theta\lambda_t - \delta)E_t & \text{if } (E_t, \Omega_t) \in \Gamma \\ (1 + r_t^M - \delta)E_t & \text{if } (E_t, \Omega_t) \in \mathbb{R}_+^2 \setminus \Gamma \end{cases}$$

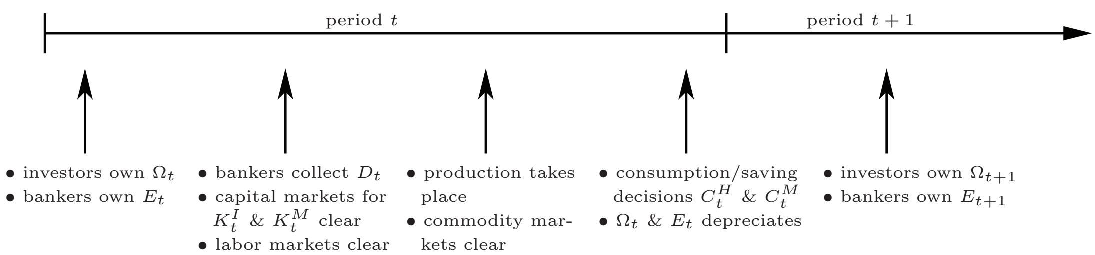
$$E_0 \text{ given,}$$

where $\beta_H = \frac{1}{1+\rho_H}$ $(0 < \beta_H < 1)$ denotes the discount factor and $\rho_H$ the discount rate.

### 3.4 Sequence of Events

The sequence of events within a specific period is depicted in Figure 2. At the beginning of period $t$, investors and bankers own $\Omega_t$ and $E_t$ units of wealth, respectively. After investors have chosen their portfolio of bonds $B_t$ and deposits $D_t$, bankers choose their investment, given their current endowment of loanable funds $E_t + D_t$. Factor markets clear and production takes place. We note that market clearing in the bond market yields $K_t^M = B_t$ and market clearing in the loan market yields $K_t^I = E_t + D_t$. We further denote $K_t = K_t^M + K_t^I$. After production factors and depositors got paid, investors, workers, and bankers consume, and commodity markets clear. Capital depreciates and evolves according to the investor's and banker's saving decision $\Omega_{t+1}(E_t, \Omega_t)$ and $E_{t+1}(E_t, \Omega_t)$.

Figure 2: Sequence of Events



# 4 Sequential Markets Equilibrium

In this section, we characterize the sequential markets equilibrium defined as follows:

**Definition 1.** *For any given* $(E_0, \Omega_0) \in \mathbb{R}^2_+$*, a sequential markets equilibrium is a sequence of factor allocations* $\{K_t^M, K_t^I, L_t^M, L_t^I\}_{t=0}^{\infty}$*, factor prices* $\{w_t^M, w_t^I, r_t^M, r_t^I\}_{t=0}^{\infty}$*, consumption choices* $\{C_t^H, C_t^B\}_{t=0}^{\infty}$*, and wealth allocations* $\{E_{t+1}, \Omega_{t+1}\}_{t=0}^{\infty}$ *such that*

1. *Given* $\Omega_0$ *and* $\{r_t^M\}_{t=0}^{\infty}$*, the allocation* $\{C_t^H, \Omega_{t+1}\}_{t=0}^{\infty}$ *solves the representative investor's utility maximization problem* (2)*.*

2. *Given* $E_0$ *and* $\{r_t^M, r_t^I\}_{t=0}^{\infty}$*, the allocation* $\{C_t^B, E_{t+1}\}_{t=0}^{\infty}$ *solves the representative banker's utility maximization problem* (7)*.*

3. *Given* $\{w_t^M, w_t^I, r_t^M, r_t^I\}_{t=0}^{\infty}$*, the allocation* $\{K_t^M, K_t^I, L_t^M, L_t^I\}_{t=0}^{\infty}$ *solves the representative firms' profit maximization problem* (1)*.*

4. *Factor markets and good markets clear.*

We split the analysis of the sequential markets equilibrium into two steps. In the first step, we characterize the intraperiod factor allocation $(K_t^M, K_t^I, L_t^M, L_t^I)$, equilibrium factor prices $(w_t^M, w_t^I, r_t^M, r_t^I)$, and the ensuing leverage $\lambda_t$ for any given beginning-of-period allocation of bank equity and investor wealth $(E_t, \Omega_t)$. In the second step, we characterize the consumption-saving policies for given beginning-of-period wealth allocation and equilibrium factor price, which finally governs the evolution of bank equity $E_{t+1}(E_t, \Omega_t)$ and investor wealth $\Omega_{t+1}(E_t, \Omega_t)$.

## 4.1 Intraperiod Equilibrium

Consider a typical period $t$ with beginning-of-period capital allocation $(E_t, \Omega_t)$. The firms' profit maximization problems given in (1) yield the usual marginal product conditions on competitive markets. Interest and wage rates satisfy

$$r_t^j(K_t^j) = \alpha z^j A_t \left(\frac{K_t^j}{L^j}\right)^{\alpha-1}, \quad j \in \{M, I\} \tag{8}$$

$$w_t^j(K_t^j) = (1-\alpha) z^j A_t \left(\frac{K_t^j}{L^j}\right)^{\alpha}, \quad j \in \{M, I\}, \tag{9}$$

where we already imposed market clearing on the labor market, i.e. $L_t^M = L^M$ and $L_t^I = L^I$. We distinguish two cases: first, the case when financial frictions are irrelevant (non-binding) and, second, the case when financial frictions are relevant (binding).

### 4.1.1 Irrelevant Financial Frictions

Suppose equity is relatively abundant, i.e. $(E_t, \Omega_t) \in \mathbb{R}^2_+ \setminus \Gamma$. Bankers hold sufficient loanable funds, such that production in sector $I$ is not limited by loan supply. In this case, financial frictions are irrelevant and competitive capital markets align interest rates in both sectors. Defining $z \doteq (\frac{z^I}{z^M})^{\frac{1}{1-\alpha}}$ and $\ell \doteq \frac{L^I}{L^M}$, condition $r_t^I(K_t^I) = r_t^M(K_t^M)$ and equation (8) yields $K_t^I = z\ell K_t^M$. In combination with the aggregate resource constraint, this condition yields

$$K_t^{M*} = \frac{\Omega_t + E_t}{1 + z\ell} = \frac{1}{1 + z\ell} K_t$$
$$K_t^{I*} = z\ell \frac{\Omega_t + E_t}{1 + z\ell} = \frac{z\ell}{1 + z\ell} K_t.$$

Incentive compatibility of the deposit contract requires that net earnings $(1 + r_t^M)E_t$ of the banker are at least as large as the non-pledgeable part of revenues $\theta K_t^I$. Therefore,

$$E_t \geq \frac{\theta K_t^I}{(1 + r_t^M(K_t^{M*}))} = \theta \frac{z\ell}{(1 + z\ell)(1 + r_t^M(K_t^{M*}))} K_t \doteq \bar{E}(K_t), \tag{10}$$

where $\bar{E}(K_t)$ denotes the lower bound of bank equity that makes financial frictions irrelevant given some overall capital $K_t = E_t + \Omega_t$ in the economy. Note that condition $E_t \geq \bar{E}(K_t)$ is an implicit characterization of the partition $(E_t, \Omega_t) \in \mathbb{R}^2_+ \setminus \Gamma$ of the state space.

### 4.1.2 Relevant Financial Frictions

Suppose equity is relatively scarce, i.e. $(E_t, \Omega_t) \in \Gamma$. Incentive compatibility of the deposit contract limits the amount of loanable funds, such that production in sector $I$ is limited by a shortage in loan supply. In this case, financial frictions are relevant. Rewriting the leverage condition (6) as $\lambda_t(r_t^M(K_t^M) - r_t^I(K_t^I) + \theta) - (1 + r_t^M(K_t^M)) = 0$, and defining the left hand side as auxiliary function $\varphi(\lambda_t)$ yields

$$\varphi(\lambda_t) \doteq r_t^M(\Omega_t + E_t - \lambda_t E_t)(\lambda_t - 1) - r_t^I(\lambda_t E_t)\lambda_t + \lambda_t \theta - 1 = 0. \tag{11}$$

For any given $(E_t, \Omega_t) \in \Gamma$, condition (11) is one equation in one unknown: equilibrium leverage $\lambda_t^*$.

The function $\varphi(\lambda_t)$ is continuous and monotonically increasing. Because financial frictions are relevant, the interest rate in sector $I$ exceeds the interest rate in sector $M$, which implies $K_t^I < z\ell K_t^M$ and $K_t^I = \lambda_t E_t$. In combination with the aggregate resource constraint, $K_t^M + K_t^I = \Omega_t + E_t$, these conditions yield an upper bound for bank leverage,

$\frac{z\ell}{1+z\ell}\frac{K_t}{E_t} > 1$, where the qualification follows from $E_t < \bar{E}(K_t)$. Suppose $\lambda_t \in [1, \frac{z\ell}{1+z\ell}\frac{K_t}{E_t}]$. Evaluating $\varphi(\lambda)$ at the lower bound of the interval gives $\varphi(1) = -(1 + r_t^I - \theta) < 0$. At the upper bound of the interval, financial frictions cease to be binding and interest rates converge. In this case,

$$\varphi\Big(\frac{z\ell}{1+z\ell}\frac{K_t}{E_t}\Big) = \frac{z\ell}{1+z\ell}\frac{K_t}{E_t}\theta - \Big(1 + r_t^M\Big(\frac{K_t}{1+z\ell}\Big)\Big).$$

Note that $\varphi(\frac{z\ell}{1+z\ell}\frac{K_t}{E_t})$ is decreasing in $E_t$ and attains zero when $E_t = \bar{E}(K_t)$. Because financial frictions are relevant, $E_t < \bar{E}(K_t)$ such that $\varphi(\frac{z\ell}{1+z\ell}\frac{K_t}{E_t}) > 0$. Therefore, by the intermediate value theorem, there exists a unique $\lambda_t^* \in [1, \frac{z\ell}{1+z\ell}\frac{K_t}{E_t}]$ satisfying $\varphi(\lambda_t^*) = 0$. The equilibrium factor allocations are then computed as follows:

$$\begin{aligned} K_t^{M*} &= K_t - \lambda_t^* E_t = \Omega_t - (\lambda_t^* - 1)E_t \\ K_t^{I*} &= \lambda_t^* E_t. \end{aligned}$$

#### 4.1.3 Existence and Uniqueness of Intraperiod Equilibrium

**Proposition 1** (Intraperiod Equilibrium: Factor Allocation)**.**
*For all pairs $(E_t, K_t)$ with $0 < E_t < K_t$, there exists a unique intraperiod equilibrium.*

*(i) If $E_t \geq \bar{E}(K_t)$, financial frictions do not matter. The capital allocation is given by $K_t^{M*} = \frac{1}{1+z\ell}K_t$ and $K_t^{I*} = \frac{z\ell}{1+z\ell}K_t$.*

*(ii) If $E_t < \bar{E}(K_t)$, financial frictions matter. The bank leverage $\lambda_t^*$ is the solution to $\varphi(\lambda^*) = 0$ and the capital allocation is given by $K_t^{M*} = \Omega_t - (\lambda_t^* - 1)E_t$ and $K_t^{I*} = \lambda_t^* E_t$.*

*Proof.* The proof directly follows from our discussion in Sections 4.1.1 and 4.1.2. □

#### 4.1.4 Comparative Statics when Financial Frictions are Relevant

We now discuss the impact of shocks to productivities, investor wealth, bank equity, and financial frictions on bank leverage, bond finance, loan finance, and output. While there is a clear and straightforward interpretation of shocks to investor wealth as an unexpected depreciation of the investor's asset holdings, the notion of shocks to bank equity requires some additional explanation. Typically, bank equity is the residual of the bank's assets and liabilities, and a bank equity shock has to be traced back to a shock to either bank assets or bank liabilities. For instance, when the actual loan default rate deviates from the expected one, bank asset holdings adjust and so does bank equity. In our model, bank equity is the banker's net worth and the bank's working capital. In this context, a shock to bank equity can be the result of risky investments that affect the return on equity but are outside of our model. We abstract from the specific source of bank equity shocks and restrict our analysis to direct changes in bank equity, without loss of generality.

Corollary 1 summarizes the impact of shocks to productivities, investor wealth, bank equity, and financial frictions on bank leverage.

**Corollary 1.** *Suppose that financial frictions matter, i.e.* $(E_t, \Omega_t) \in \Gamma$. *Then, bank leverage* $\lambda_t$

*(i) increases in* $A_t$ *and* $z^I$ *and decreases in* $z^M$*,*

*(ii) increases in* $\Omega_t$ *and decreases in* $E_t$*, and*

*(iii) decreases in* $\theta$*.*

*Proof.* See Appendix A.1. □

The main intuition for the results can be derived from comparing the profits of a single bank if it complies with the deposit contract, $(1 + r_t^I)k_t^I - (1 + r_t^M)d_t$, to the profits of defecting, $\theta k_t^I$. The intuitive argument neglects some equilibrium adjustments which, however, only partially off-set the described effect.

First, a productivity increase in sector $M$ *ceteris paribus* increases the deposit rate and thus the repayment obligation that arises from the deposit contract, $(1 + r_t^M)d_t$. Profits from complying with the deposit contract decline, and investors have to cut down their investment into deposits to preserve the incentive compatibility of the deposit contract. Thus, bank leverage decreases. A productivity increase in sector $I$ *ceteris paribus* increases the revenues from providing loans to sector $I$, $(1 + r_t^I)k_t^I$, and thus profits from complying with the deposit contract. Investors can thus increase their deposits without interfering with the incentive compatibility of the deposit contract. As a result, bank leverage increases. The effect of a common productivity shock is more involved, as it *ceteris paribus* increases the banks' revenues from investing into sector $I$ as well as the repayment obligation to depositors. However, because $r_t^I > r_t^M$, the effect on the revenues dominates the effect on the repayment obligation, such that similar to the productivity shock in sector $I$, bank leverage increases.

Second, an increase in aggregate investor wealth $\Omega_t$ *ceteris paribus* increases investment in sector $M$ and thus decreases $r_t^M$. Therefore, the bank's repayment obligation from complying with the deposit contract declines and profit increases. As it becomes easier for investors to induce incentive compatible behavior, bank leverage increases. An increase in aggregate bank equity $E_t$ *ceteris paribus* increases both bond finance $K_t^M$ and loan finance $K_t^I$. Interest rates fall in both sectors, such that the bank's revenues from investing into sector $I$ and the repayment obligation to depositors decrease. Because loan finance is more elastic to changes in the equity stock than bond finance,[19] the effect on revenues dominates the effect on the repayment obligation, such that profits from complying with the deposit contract fall. As a result, investors have to reduce their deposits in order to restore incentive compatibility such that bank leverage decreases.

Finally, when financial frictions between depositors and banks become more severe, the value of each bank's outside option from defecting increases. Investors cut down their investment in deposits to incentivize banks to comply with the deposit contract. As a result, leverage declines.

The following corollary establishes the impact of shocks to common productivity, investor wealth, bank equity, and financial frictions on investments in the two sectors.

**Corollary 2.** *Suppose that financial frictions matter, i.e.* $(E_t, \Omega_t) \in \Gamma$. *Then,*

*(i)* $K_t^I$ *increases in* $A_t$ *and* $K_t^M$ *decreases in* $A_t$,

*(ii.1)* $K_t^I$ *and* $K_t^M$ *increase in* $\Omega_t$,

*(ii.2)* $K_t^I$ *increases in* $E_t$ *and* $K_t^M$ *decreases in* $E_t$, *and*

*(iii)* $K_t^I$ *decreases in* $\theta$ *and* $K_t^M$ *increases in* $\theta$.

*Proof.* See Appendix A.2. □

The responses of leverage, bond finance, and loan finance to downturns resulting from a negative shock to common productivity, a decline in bank equity capital, or a worsening of financial frictions – or any combination thereof, established in Corollaries 1 and 2, are consistent with two empirical facts: First, book leverage in the banking sector is procyclical, because $\frac{\partial \lambda_t}{\partial A_t} > 0$, $\frac{\partial \lambda_t}{\partial E_t} > 0$, and $\frac{\partial \lambda_t}{\partial \theta} < 0$ – a pattern that is well documented e.g. in Adrian and Shin (2014), Adrian and Boyarchenko (2012), Adrian and Boyarchenko (2013), and Nuño and Thomas (2012). Second, because $\frac{\partial K_t^I}{\partial A_t} > 0$, $\frac{\partial K_t^I}{\partial E_t} > 0$, and $\frac{\partial K_t^I}{\partial \theta} < 0$, loan finance is procyclical and because $\frac{\partial K_t^M}{\partial A_t} < 0$, $\frac{\partial K_t^M}{\partial E_t} < 0$, and $\frac{\partial K_t^M}{\partial \theta} > 0$, bond finance

[19] The elasticity of loan finance with respect to equity is $\frac{\partial K_t^I}{\partial E_t} \frac{E_t}{K_t^I} = 1$ whereas the elasticity of bond finance with respect to equity is $\frac{\partial K_t^M}{\partial E_t} \frac{E_t}{K_t^M} = \frac{(\lambda_t - 1)E_t}{\Omega_t + (\lambda_t - 1)E_t} < 1$.

is countercyclical. Thus, the bond-to-loan finance ratio is countercyclical – see De Fiore and Uhlig (2011) and De Fiore and Uhlig (2015).

Finally, we establish the impact of shocks to common productivity, investor wealth, and bank equity on total output.

**Corollary 3.** *Suppose that financial frictions matter, i.e.* $(E_t, \Omega_t) \in \Gamma$. *Then, total output* $Y_t$

*(i) increases in* $A_t$*,*

*(ii) increases in* $\Omega_t$ *and* $E_t$*, and*

*(iii) decreases in* $\theta$*.*

*Proof.* See Appendix A.3. □

An increase in productivity or total capital, i.e. either investor wealth or bank equity capital, directly rises total output. For an increase in the financial friction, we note that this leads to a more inefficient allocation of capital and, thus, has a negative impact on total output.

The key comparative statics of Corollaries 1 to 3 are summarized in Table 1.

Table 1: Comparative Statics

| | leverage $\lambda$ | loans $K^I$ | bonds $K^M$ | output $Y$ |
|---|---|---|---|---|
| productivity ($\Delta A > 0$) | + | + | − | + |
| investor wealth ($\Delta\Omega > 0$) | + | + | + | + |
| bank equity ($\Delta E > 0$) | − | + | − | + |
| financial friction ($\Delta\theta > 0$) | − | − | + | − |

## 4.2 Intertemporal Consumption-Saving Decision

Because bankers and investors have logarithmic utility and their disposable income is linear homogenous in wealth, their consumption-saving policies are linear homogenous in wealth, too. In fact, bankers and investors save a constant fraction of their end-of-period net worth.

**Proposition 2** (Intertemporal Equilibrium: Consumption and Saving)**.**
*Given* $(E_t, \Omega_t) \in \mathbb{R}^2_+$ *and given* $r^M_t(E_t, \Omega_t)$, $r^I_t(E_t, \Omega_t)$, *and* $\lambda_t(E_t, \Omega_t)$ *from the factor allocation characterized in Proposition 1, the banker's and investor's consumption-saving policies are linear homogenous in end-of-period net worth.*

*(i) The consumption-saving policy functions*

$$
\begin{aligned}
C^B_t &= (1-\beta_B)(1 + r^B_t(E_t, \Omega_t))E_t \\
E_{t+1} &= \beta_B(1 + r^B_t(E_t, \Omega_t))E_t
\end{aligned}
$$

*solve the banker's utility maximization problem* (7) *where* $r^B_t(E_t, \Omega_t)$ *is the (net) return on equity in period* $t$ *given by*

$$
r^B_t(E_t, \Omega_t) \doteq \begin{cases} \theta\lambda_t(E_t, \Omega_t) - \delta - 1 & \textit{if} \quad (E_t, \Omega_t) \in \Gamma \\ r^M_t(E_t, \Omega_t) - \delta & \textit{if} \quad (E_t, \Omega_t) \in \mathbb{R}^2_+ \setminus \Gamma. \end{cases}
$$

*(ii) The consumption-saving policy functions*

$$
\begin{aligned}
C^H_t &= (1-\beta_H)(1 + r^M_t(E_t, \Omega_t) - \delta)\Omega_t \\
\Omega_{t+1} &= \beta_H(1 + r^M_t(E_t, \Omega_t) - \delta)\Omega_t.
\end{aligned}
$$

*solve the investor's utility maximization problem* (2)*.*

*Proof.* See Appendix A.4. □

Using Condition (11) to rewrite the (net) return on equity for the case in which frictions are binding,

$$
\begin{aligned}
r^B_t(E_t, \Omega_t) &= \theta\lambda_t(E_t, \Omega_t) - \delta - 1 \\
&= \lambda_t(1 + r^I_t(E_t, \Omega_t)) - (\lambda_t(E_t, \Omega_t) - 1)(1 + r^M_t(E_t, \Omega_t)) - \delta - 1 \\
&= r^M_t(E_t, \Omega_t) + \lambda_t(E_t, \Omega_t)(r^I_t(E_t, \Omega_t) - r^M_t(E_t, \Omega_t)) - \delta,
\end{aligned}
$$

reveals that banks benefit from the interest rate spread and from higher bank leverage. For the remainder of this paper, we will assume that bankers are more impatient than investors, i.e. $\beta_B < \beta_H$ or $\rho_B > \rho_H$. It is important to stress that the assumption on preferences reflects a more fundamental (and more complex) capital cost argument that leads to relative scarcity of bank equity capital. The opposite assumption would be strongly counterfactual given the experience with very low levels of bank equity capital over the last decades.

# 5 Steady State

In this section, we characterize the steady state allocation, prove its existence and uniqueness, and analyze how permanent changes in the financial friction and technological progress affect the steady state allocation.

## 5.1 Existence and Uniqueness of the Steady State

In a steady state, allocations and prices are constant across time. Suppose that the economy is in a steady state in which financial frictions are relevant. Setting $E_{t+1} = E_t$ and $\Omega_{t+1} = \Omega_t$, the saving policies in Proposition 2 yield

$$\hat{r}^M = \delta + \rho_H \tag{12}$$

$$\hat{\lambda} = \frac{\delta + \rho_B + 1}{\theta}. \tag{13}$$

where $\hat{x}$ denotes the steady state value of variable $x$. Combining the definition of bank leverage, Equation (6), with Equations (12) and (13) yields

$$\hat{r}^I = \hat{r}^M + \frac{\theta(\rho_B - \rho_H)}{1 + \delta + \rho_B} = \delta + \rho_H + \frac{\theta(\rho_B - \rho_H)}{1 + \delta + \rho_B}. \tag{14}$$

Because $\rho_B > \rho_H$, the interest rates satisfy $\hat{r}^I > \hat{r}^M$, which is consistent with the presupposition of binding financial frictions. Given $\hat{r}^I$ and $\hat{r}^M$, the steady state factor and wealth allocations compute as

$$\hat{K}^M = \left(\frac{\alpha z^M A}{\hat{r}^M}\right)^{\frac{1}{1-\alpha}} L^M \tag{15}$$

$$\hat{K}^I = \left(\frac{\alpha z^I A}{\hat{r}^I}\right)^{\frac{1}{1-\alpha}} L^I \tag{16}$$

$$\hat{E} = \left(\frac{\alpha z^I A}{\hat{r}^I}\right)^{\frac{1}{1-\alpha}} \frac{\theta}{1 + \delta + \rho_B} L^I \tag{17}$$

$$\hat{\Omega} = \hat{K}^M_t + \hat{K}^I_t - \hat{E}. \tag{18}$$

So far we have assumed that financial frictions matter in the steady state. We next show that there does not exist a steady state in which financial frictions are irrelevant. Suppose that at the steady state, financial frictions are irrelevant, i.e. $(E_t, \Omega_t) \in \mathbb{R}^2_+ \setminus \Gamma$. According to Proposition 2, capital accumulation is governed by $E_{t+1} = \beta_B(1+r^M_t - \delta)E_t$ and $\Omega_{t+1} = \beta_H(1 + r^M_t - \delta)\Omega_t$. Recalling that $\beta_B < \beta_H$, we note that first, if $\Omega_{t+1} = \Omega_t$, bank equity decreases and, second, if $E_{t+1} = E_t$, investor wealth increases. Taken together, this contradicts the presupposition that there is a steady state in which financial frictions are irrelevant.

**Proposition 3** (Existence and Uniqueness of the Steady State)**.**
*There exists a unique steady state* $(\hat{E}, \hat{\Omega})$. *Financial frictions are binding and allocations are given by Equations* (12) *to* (18)*.*

*Proof.* The proof directly follows from the preceding discussion. □

## 5.2 Impact of Financial Frictions and Technological Progress on the Steady State

First, a permanent increase in financial frictions, i.e. a permanent shift in the belief in the bank's repayment behavior, has several implications for the steady state allocation, as the inefficiency of the allocation increases. From Proposition 3, we derive the following corollary

**Corollary 4.** *An increase of the intensity of financial frictions, i.e. an increase of* $\theta$*,*

*(i) lowers the steady-state level of capital* $\hat{K}$*, and*

*(ii) increases bank equity* $\hat{E}$ *if bankers are not too impatient.*

*Proof.* The statement for $\hat{K}$ follows immediately from the fact that a higher value of $\theta$ increases $\hat{r}^I$, which leads to a reduction in $\hat{K}^I$. At the same time, $\hat{r}^M$ is unaffected by the degree of the financial friction, such that $\hat{K}^M$ is unaffected. Therefore, $\hat{K} = \hat{K}^I + \hat{K}^M$ falls. The impact on $\hat{E}$ is more involved. Differentiation yields

$$\frac{\partial \hat{E}}{\partial \theta} = \frac{1}{1+\delta+\rho_B}\left\{\frac{1}{\hat{r}^I}\left(\frac{\alpha A z^I}{\hat{r}^I}\right)^{\frac{1}{1-\alpha}}\left(\hat{r}^M - \frac{\alpha}{1-\alpha}\frac{\theta(\rho_B-\rho_H)}{1+\delta+\rho_B}\right)\right\}L^I. \tag{19}$$

When $\rho_B$ is sufficiently close to $\rho_H$, we get $\frac{\partial \hat{E}}{\partial \theta} > 0$. □

An important consequence of Corollary 4 is that, in the steady state, more severe financial frictions lower the total amount of capital and the share owned by investors, but not the wealth of bankers if bankers are not too impatient. The reason is subtle. A higher value of $\theta$ lowers leverage. However, when $\rho_B$ is close to $\rho_H$, a steady state requires that $\hat{r}^I$ is close to $\hat{r}^M$ and thus $\hat{K}^I$ is close to $\frac{\hat{K}^M}{z\ell}$. As the latter is independent of $\theta$, variations of $\theta$ have little effect on $\hat{K}^I$ for $\rho_B$ close to $\rho_H$. Because $\hat{K}^I = \hat{\lambda}\hat{E}$, a higher value of $\theta$ is associated with a higher value of $\hat{E}$.

Second, consider a permanent increase in the common factor productivity. Conditions (12) to (14) directly reveal that steady state interest rates and leverage are independent of the technology level, and the capital allocations and wealth distributions are proportional to $A^{1/(1-\alpha)}$. The following corollary summarizes these considerations:

**Corollary 5.** *An increase in common total factor productivity by* $(1+\Delta_A)$ *yields an increase of the steady state capital allocation and wealth distribution by factor* $(1+\Delta_A)^{1/(1-\alpha)}$. *The bond-to-loan finance ratio is independent of changes in common total factor productivity.*

*Proof.* The proof directly follows from Proposition 3. □

# 6 Stability, Dynamics, and Leverage as Automatic Stabilizer

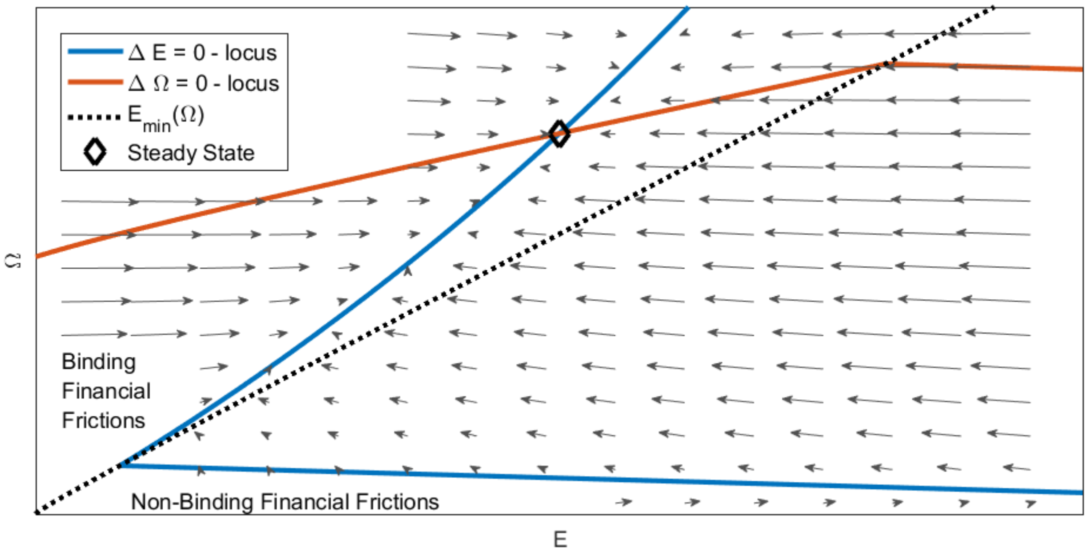
This section characterizes global dynamics and establishes global stability of the economy. We provide new insights into the propagation of shocks and show that the elasticity of bank leverage with respect to bank equity is an essential factor for the resilience of the economy to adverse shocks affecting bank balance sheets. This section concludes with a brief discussion of dynamic responses to permanent shocks to productivity and the financial friction.

## 6.1 Global Stability

To establish global stability, our analysis proceeds in two steps. In the first step, we show that for any initial $(E_0, \Omega_0) \in \mathbb{R}^2_+ \setminus \Gamma$, i.e. for any initial capital allocation for which financial frictions are irrelevant (non-binding), the economy converges to the partition in the state space in which frictions become binding in finite time $\tau > 0$. In the second step, we show that for any $(E_\tau, \Omega_\tau) \in \Gamma$, i.e. for any capital allocation for which financial frictions are relevant (binding), the economy converges to its unique steady state. The global dynamics are depicted in the phase diagram, Figure 3.

The dotted line in Figure 3 represents Equation (10) and separates $\mathbb{R}^2_+$ in the two regions in which financial functions are relevant (north-west) and irrelevant (south-east), respectively. First, consider an equity-wealth allocation for which financial frictions are irrelevant, i.e. $(E_0, \Omega_0) \in \mathbb{R}^2_+ \setminus \Gamma$ or, equivalently, $\frac{E_0}{K_0} \geq \theta \frac{z\ell}{1+z\ell} \frac{1}{1+r_0^M(K_0)}$. In this case, equity is relatively abundant and the allocation $(E_0, \Omega_0)$ is south-east of the dotted line in the phase diagram. Suppose now that financial frictions remain irrelevant in all future periods. Then, the law of motions for bank equity and investor wealth (see Proposition

Figure 3: Phase Diagram




2) imply that the equity-to-wealth ratio $\frac{E_t}{\Omega_t}$ declines at a constant rate $\frac{\beta_H - \beta_B}{\beta_H} > 0$. Moreover, $\frac{E_t}{K_t}$ declines at an accelerating rate $\frac{\beta_H - \beta_B}{\beta_B E_t/\Omega_t + \beta_H} \geq \frac{\beta_H - \beta_B}{\beta_B E_0/\Omega_0 + \beta_H} > 0$, such that $\lim_{t\to\infty} \frac{E_t}{K_t} = 0$. We note that because the production technologies satisfy the Inada conditions, there is a strictly positive lower bound for the series of total capital $\{K_t\}_{t=0}^{\infty}$ for any $(E_0, \Omega_0)$. Specifically, for $K_t$ sufficiently low, the capital return $r_t^M(K_t)$ is sufficiently high to spur the accumulation of investor wealth and bank equity capital. As a result, there exists a $\tau$ such that

$$\frac{E_\tau}{K_\tau} < \theta \frac{z\ell}{1 + z\ell} \frac{1}{1 + r_t^M(K_\tau)},$$

which contradicts the presupposition that financial frictions remain irrelevant in all future periods. Therefore, financial frictions become binding in finite time.

Second, consider an allocation $(E_\tau, \Omega_\tau) \in \Gamma$, i.e. financial frictions are relevant. In this case, equity is relatively scarce, which corresponds to the partition in the phase diagram north-west of the dotted line. The $\Delta E = 0$-locus is the combination of all $E$ and $\Omega$ such that $E_{t+1} = E_t$. According to Proposition 2, $E_{t+1} = E_t$ corresponds to $1 = \beta_B(1 + r_t^B(E_t, \Omega_t))$. Implicit differentiation of the $\Delta E = 0$-locus condition yields $\frac{\partial \Omega}{\partial E}|_{\Delta E = 0} = -\frac{\partial \lambda}{\partial E} / \frac{\partial \lambda}{\partial \Omega} > 0$, i.e. the $\Delta E = 0$-locus has a positive slope. On the left side of the locus, equity increases, and on the right side, equity decreases. In a similar vein, the $\Delta\Omega = 0$-locus is the combination of all $E$ and $\Omega$ such that $\Omega_{t+1} = \Omega_t$. According to Proposition 2, $\Omega_{t+1} = \Omega_t$ corresponds to $1 = \beta_H(1 + r_t^M(E_t, \Omega_t) - \delta)$. Implicit differentiation of the $\Delta\Omega = 0$-locus condition yields $\frac{\partial\Omega}{\partial E}|_{\Delta\Omega=0} = -(1-\lambda-\frac{\partial\lambda}{\partial E}E)/(1-\frac{\partial\lambda}{\partial\Omega}E) = -\frac{\partial K^M}{\partial E}/\frac{\partial K^M}{\partial\Omega} > 0$. Above the locus, investor wealth decreases and below the locus, investor wealth increases. We further note that for $(E,\Omega) \in \Gamma$,

$$\begin{aligned}
\frac{\partial\Omega}{\partial E}|_{\Delta E=0} - \frac{\partial\Omega}{\partial E}|_{\Delta\Omega=0} &= -\frac{\frac{\partial\lambda}{\partial E}}{\frac{\partial\lambda}{\partial\Omega}} + \frac{1-\lambda-\frac{\partial\lambda}{\partial E}E}{1-\frac{\partial\lambda}{\partial\Omega}E} = \frac{-\frac{\partial\lambda}{\partial E}+(1-\lambda)\frac{\partial\lambda}{\partial\Omega}}{\frac{\partial\lambda}{\partial\Omega}(1-\frac{\partial\lambda}{\partial\Omega}E)} \\
&= \frac{\frac{\partial\varphi}{\partial E}-(1-\lambda)\frac{\partial\varphi}{\partial\Omega}}{\frac{\partial\varphi}{\partial\lambda}\frac{\partial\lambda}{\partial\Omega}(1-\frac{\partial\lambda}{\partial\Omega}E)} = \frac{-\frac{\partial r^M}{\partial K^M}(\lambda-1)^2-\frac{\partial r^I}{\partial K^I}\lambda^2+\frac{\partial r^M}{\partial K^M}(\lambda-1)^2}{\frac{\partial\varphi}{\partial\lambda}\frac{\partial\lambda}{\partial\Omega}(1-\frac{\partial\lambda}{\partial\Omega}E)} \\
&= \frac{-\frac{\partial r^I}{\partial K^I}\lambda^2}{\frac{\partial\varphi}{\partial\lambda}\frac{\partial\lambda}{\partial\Omega}(1-\frac{\partial\lambda}{\partial\Omega}E)} > 0, \qquad (20)
\end{aligned}$$

i.e. the $\Delta E = 0$-locus is steeper than the $\Delta\Omega = 0$-locus. Inspecting the relative location of the loci and the dynamics of bank equity and investor wealth relative to the loci, the phase diagram reveals stability of the economic system for any $(E_\tau, \Omega_\tau) \in \Gamma$.

Summarizing both observations yields the following proposition:

**Proposition 4** (Global Stability of the Steady State with Financial Frictions)**.**
*For any initial $(E,\Omega) \in \mathbb{R}^2_+$, the economy converges to the unique steady state in which financial frictions matter.*

*Proof.* The proof directly follows from the previous discussion. □

### 6.2 Dynamics and Leverage as Automatic Stabilizer

We now confine attention to economic dynamics in response to capital shocks to further investigate the general economic dynamics for any $(E_\tau, \Omega_\tau) \in \Gamma$.

First, consider a negative shock to investor wealth $\Omega_t$ that hits the economy in its steady state. According to Corollaries 1 and 2, there is an immediate decrease in both, bond and loan finance which leads to an increase in $r_t^M$ such that the growth rate of investor wealth increases relative to its steady state value. At the same time, Corollary 1 reveals that bank leverage decreases which means that the growth rate of bank equity falls short of its steady state value. Thus, while the growth rate of investor wealth already starts to increase and puts investor wealth on a recovery path, the induced decline in bank equity capital decreases bank profits, and next period equity holdings. This, in turn, lowers the capacity to attract loanable funds in the subsequent periods. The decline in investor wealth triggers a persistent misallocation towards the less capital efficient sector $M$. While this mechanism can be active for several periods, Proposition 4 implies that there must be a turning point at which equity is sufficiently scarce to raise leverage above its steady state value. Then, bank equity rebounds and the economy converges to its steady state.

Second, consider a negative shock to bank equity $E_t$. According to Corollaries 1 and 2, investors reallocate their funds towards bond finance such that $r_t^M$ decreases and investor wealth starts to decline, i.e. there is a transmission of the bank equity shock to investor wealth. At the same time, bank leverage increases as the profit margin for banks increases when the deposit rate falls and it becomes easier to incentivize bankers to keep to the contractual obligations of the deposit contract. As a result, the growth rate of bank equity increases. This mechanism already partially compensates the initial decline in bank equity and therefore buffers the resource reallocation towards the less capital efficient sector: the response of bank leverage helps to stabilize the economy. Nevertheless, next period bank equity holdings are still below their steady state value, which affects bank profits and the capacity to attract loanable funds in the subsequent periods. Because of global stability (see Proposition 4), there must be a turning point at which investor wealth rebounds, its growth rate overshoots, and the economy converges to its steady state.

Inspecting the mechanism that underlies the propagation of the shock to bank equity delivers novel insights. As bank equity declines, loan finance declines *ceteris paribus*. However, because of $\frac{\partial \lambda_t}{\partial E_t} < 0$, the decline in bank equity is accompanied by an increase in bank leverage, which already counteracts the direct effects on loan finance, bank profits and the capacity to attract loanable funds in the subsequent period. Essentially, the stronger the counter-reaction of bank leverage, the easier it is for the economy to absorb adverse shocks to bank equity. This is because it avoids triggering, or at least contributes to buffering, the persistent and potentially decline of bank finance due to lower bank equity capital – which is often referred to as the bank capital transmission channel.

The sensitivity of bank leverage and thus the automatic stabilization mechanism depends *inter alia* on financial institutions, e.g. capital regulation, and labor market institutions, e.g. employment protection legislation. First, capital regulation imposes an upper limit on bank leverage beyond which there is no further adjustment possible. While this weakens the automatic stabilization through leverage adjustment, capital regulation can help to push down the initial shock size by limiting the multiplier effect of leverage at first place. Second, when labor mobility is high, there is an immediate reallocation of production factors in response to an adverse shock to bank equity, leaving the capital-to-output ratios in both sectors almost unaffected. Therefore, interest rates are only mildly affected and so is bank leverage. Essentially, while labor reallocation provides a different channel through which the economy absorbs adverse shocks affecting the bank balance sheet, it also leads to a persistent sectoral shift towards the less capital efficient bond financed industries: recovery in the banking sector slows down. In order to assess the importance of labor mobility for the resilience of the financial system to adverse shocks, we compare the results in this paper with a version of the model discussed in Gersbach et al. (2016) in which labor is perfectly mobile between both sectors in Gersbach et al. (2016). We find that, *mutatis mutandis*, shocks are substantially more persistent as leverage is insensitive to capital reallocation. Because persistent shocks are in general more severe in terms of welfare losses than comparable transitory shocks, the novel feedback channel – from labor market institutions to the performance of the financial system – can have substantial welfare implications. Therefore, a judicious choice of labor market institutions can help to stabilize the financial system from both, an *ex-ante* and *ex-post* perspective. Note that while we consider capital regulation quantitatively in Section 8 and theoretically in Appendix C, the discussion of labor market institutions is beyond the scope of this paper.

### 6.3 Permanent Shocks to Productivity

Suppose the economy is at its steady state and gets hit by a negative shock to productivity in sector $M$. According to Corollary 1, bank leverage and, as a consequence, loan finance $K_t^I = \lambda_t E_t$ increase. Bank profits rise such that next period bank equity holdings exceed their steady state value. The productivity shock in sector $M$ triggers an initial boom in the banking sector. On the contrary, returns in sector $M$ decline, which implies that the growth rate of bond finance turns negative. In the long-run, however, bank leverage and loan finance return to their previous levels as their steady state values are independent of productivity levels. Therefore, the initial boom in the banking sector is accompanied by a bust in the long-run. In contrast, bond finance and investor wealth decreases permanently.

The situation is different when the economy is hit by a negative productivity shock in sector $I$. According to Corollary 1, bank leverage falls and, because initial bank equity is unaffected, loan finance decreases. As a result, the growth rate of bank equity declines. Investors shift funds from deposits to bonds which pushes down the returns in sector $M$ such that the growth rate of investor wealth falls as well. In the long-run, however, the return $r_t^M$ in sector $M$, and bond finance $K_t^M$ go back to their previous level, as their steady state value is independent of productivity in sector $I$. In contrast, loan finance and equity holdings decrease permanently.

Finally, consider a negative shock to common factor productivity. As shown in Corollaries 1 and 2, a decline in common factor productivity is accompanied by a decrease in bank leverage, a decrease in loan finance and an increase in bond finance. In essence, there is a shift towards the less capital efficient production sector with output effects amplified accordingly. The returns in both sectors decrease, which leads to a decline in the growth rate of investor wealth, and, more importantly, bank leverage decreases as well, which leads to a decline in bank equity holdings and therefore can trigger the costly bank capital transmission channel. In the long-run, bond an loan finance decline, and so does bank equity capital and investor wealth. However, bank leverage, is unaffected in the long-run.

### 6.4 Permanent Shocks to Financial Frictions

There are several examples of permanent shocks to the financial friction between depositors and bankers that could materialize in an increase in $\theta$. For instance, it can become more difficult to enforce contractual obligations thereby worsening the underlying moral hazard problem. Another example is decreasing trust in the banking sector as a result of shifted beliefs about the repayment behavior of bankers.

Consider an economy that is at its steady state $(\hat{E}(\theta), \hat{\Omega}(\theta))$, associated with some level $\theta$ of financial frictions. Suppose that the economy is hit by a permanent shock that worsens financial frictions, i.e. $\theta$ increases to $\theta'$ ($\theta' > \theta$). We will now establish an analytical result regarding the consequences for bankers of such a shock.

**Proposition 5.** *Suppose that $\rho_B$ is sufficiently close to $\rho_H$ and the economy is hit by a negative permanent shock to financial frictions ($\theta \to \theta' > \theta$). Then, the intertemporal utility of bankers after the shock is higher than in the steady state associated with $\theta$.*

*Proof.* As a direct consequence of Corollary 4, steady state bank equity increases from $\hat{E}(\theta)$ to $\hat{E}(\theta')$.[20] This means that during the transition phase, $\theta'\lambda_t$ has to be larger than $\delta + \rho_B + 1$ (see Equations (13) and (19)) and thus consumption of bankers during the transition phase is higher than in the steady state associated with $\theta$. As the steady state return on equity is independent of financial frictions, bankers will have higher utility in each period when the economy is hit by an adverse shock to financial frictions. □

In contrast to bankers, investors and workers are hurt by an increase in financial frictions. Workers are also hurt in the long-run, as aggregate wages decline towards the new steady state associated with $\theta' > \theta$. For investors, however, the intraperiod utility losses vanish over time, as the interest rate $r_t^M$ converges to $\hat{r}^M = \delta + \rho_H$, which is independent of $\theta$.

[20] Note that we do not show that the movement from $\hat{E}(\theta)$ to $\hat{E}(\theta')$ is monotonic. However, as initially $\theta'\lambda_t$ is larger than $\delta + \rho_B + 1$, a potential overshooting of bank equity above $\hat{E}(\theta')$ later on would not invalidate the conclusion.

# 7 Managing Recoveries

This section discusses macroprudential policies to manage financial and banking crises when adverse shocks affect bank equity holdings. We first focus on policies with a full set of policy instruments including consumption taxes, saving subsidies and public financed bank re-capitalization. Second, we confine attention to policies with a limited set of instruments, specifically, public financed bank re-capitalization and dividend payout restrictions.

## 7.1 Pareto-Improving Recoveries

We show that there exist Pareto-improving incentive-compatible policies that stimulate capital accumulation and accelerate economic recovery after a shock to bank equity. Specifically, we consider an equity shock at the end of period $t$, that is after the production stage and before the consumption-saving decisions are made. This timing excludes the possibility that the government can reallocate resources prior to the production stage and redistribute the benefits afterwards thereby bypassing the financial friction. Moreover, we confine attention to policies that implement a direct transfer of endowments $T_0$ from investors to banks only in initial period, i.e. directly after the shock. The set of policy instruments includes consumption taxes $\tau_t^W, \tau_t^H$, and $\tau_t^B$ for workers, investors, and bankers, respectively, and saving subsidies $\sigma_t^H$ and $\sigma_t^B$ for investors and bankers, respectively. For convenience, we start with these five instruments, but as it will be shown later, we essentially only need three instruments, $\{\tau_t^W, \tau_t^H, \tau_t^B\}$ or $\{\tau_t^W, \sigma_t^H, \sigma_t^B\}$ as consumption taxes on investors and bankers are indirect savings subsidies.

**Proposition 6** (Pareto-Improving Incentive-Compatible Budget-Neutral Policies).
*Suppose that financial frictions matter, i.e.* $(E_0, \Omega_0) \in \Gamma$, *and the economy is hit by a negative shock to bank equity after production took place and before investment and saving decisions are made. Then, there exists an investor-financed re-capitalization of banks* $T_0$ *in* $t = 0$, *and a sequence of consumption taxes* $\{\tau_t^W, \tau_t^H, \tau_t^B\}_{t=0}^{\infty}$ *and saving subsidies* $\{\sigma_t^H, \sigma_t^B\}_{t=0}^{\infty}$ *such that this policy is*

*(i) Pareto-improving,*

*(ii) incentive compatible in the sense that bankers are not encouraged to depleting bank equity excessively in the expectation of a re-capitalization, and*

*(iii) budget neutral in the sense that the government's period budget constraint is satisfied in each period.*

*Proof.* Consider shocks to bank equity $\Delta E$ after the production stage. We show that a marginal increase in $T_0$ and appropriate consumption and saving taxes are Pareto-improving and accelerate economic recovery. When shocks occur after the production stage, adjustment of the transfer scheme leave aggregate resources, i.e. the right-hand side of the aggregate resource constraint

$$\begin{aligned} C_0^B + C_0^H + C_0^W + E_1 + \Omega_1 = \\ (1-\delta)(E_0 - \Delta E + T_0) + (1-\delta)(\Omega_t - T_0) + Y_0^M(K_0^M, L^M) + Y_0^I(K_0^I, L^I) \end{aligned}$$

unaffected. Therefore, any budget feasible policy satisfies $\mathrm{d}C_0^B + \mathrm{d}C_0^H + \mathrm{d}C_0^W + \mathrm{d}E_1 + \mathrm{d}\Omega_1 = 0$ for changes in these five variables. In period 0, consumption and saving policies are (see proof of Proposition 2, Appendix A.4)

$$\begin{aligned} C_0^W &= \frac{1}{1+\tau_0^W}\left(w_0^M L^M + w_0^I L^I\right) \\ C_0^H &= \frac{1-\beta_H}{1+\tau_0^W}((1+r_0^M-\delta)\Omega_0 - T_0) \\ C_0^B &= \frac{1-\beta_B}{1+\tau_0^B}((\theta\lambda_0-\delta)E_0 - \Delta E + T_0) \\ \Omega_1 &= \frac{\beta_H}{1+\sigma_0^W}((1+r_0^M-\delta)\Omega_0 - T_0) \\ E_1 &= \frac{\beta_B}{1+\sigma_t^B}((\theta\lambda_t-\delta)E_0 - \Delta E + T_0), \end{aligned}$$

where $C_0^W$, $C_0^H$, $C_0^B$ are the consumption levels of workers, investors and bankers, respectively. Note that for any consumption and saving policy, there is an independent policy instrument that compensates for the change in investor-financed re-capitalization. Specifically, one can find consumption taxes $(\tau_0^W, \tau_0^H, \tau_0^B)$ such that $\mathrm{d}C_0^B = 0$, $\mathrm{d}C_0^H = 0$, and $\mathrm{d}C_0^W = 0$ and saving subsidies $(\sigma_0^H, \sigma_0^B)$ such that $\mathrm{d}E_1 + \mathrm{d}\Omega_1 = 0$, $\mathrm{d}E_1 > 0$, and $\mathrm{d}\Omega_1 = -\mathrm{d}E_1 < 0$. This policies leaves period 0 consumption of all agents unaffected while implementing a reallocation from investor wealth to bank equity in period 1.

The aggregate resource constraint in period 1 reads

$$C_1^B + C_1^H + C_1^W + E_2 + \Omega_2 = (1-\delta)E_1 + (1-\delta)\Omega_1 + Y_1^M(K_1^M, L^M) + Y_1^I(K_1^I, L^I).$$

We now show that while $E_1 + \Omega_1$ is unaffected by the re-allocation of endowments, there is a positive impact on total output $Y_1$:[21]

$$\begin{aligned}\frac{\partial Y_1}{\partial T_0} &= \sum_{i\in\{M,I\}} \frac{\partial Y_1^i}{\partial K_1^i}\Big(\frac{\partial K_1^i}{\partial E_1}\frac{\partial E_1}{\partial T_0} + \frac{\partial K_1^i}{\partial \Omega_1}\frac{\partial \Omega_1}{\partial T_0} + \frac{\partial K_1^i}{\partial \lambda_1}\frac{\partial \lambda_1}{\partial T_0}\Big)\\ &= r_1^M\Big( - (\lambda_1 - 1)\frac{\partial E_1}{\partial T_0} - \frac{\partial E_1}{\partial T_0} - \frac{\partial \lambda_1}{\partial T_0}E_1\Big) + r_1^I\Big(\lambda_1\frac{\partial E_1}{\partial T_0} + \frac{\partial \lambda_1}{\partial T_0}E_1\Big)\\ &= \lambda_1(r_1^I - r_1^M)\Big(\frac{\partial E_1}{\partial T_0} + \frac{\partial \lambda_1}{\partial T_0}\frac{E_1}{\lambda_1}\Big) > 0.\end{aligned}$$

The inequality follows from

$$\begin{aligned}\frac{\partial \lambda_1}{\partial T_0}\frac{E_1}{\lambda_1} = \frac{\partial E_1}{\partial T_0}\frac{\partial \lambda_1}{\partial E_1}\frac{E_1}{\lambda_1} &= -\frac{\partial E_1}{\partial T_0}\frac{-\frac{\partial r_1^M}{\partial K_1^M}\lambda_1(\lambda_1-1) - \frac{\partial r_1^I}{\partial K_1^I}\lambda_1^2}{-\frac{\partial r_1^M}{\partial K_1^M}E_1(\lambda_1-1) - \frac{\partial r_1^I}{\partial K_1^I}E_1\lambda_1 + r_t^M - r_t^I + \theta}\frac{E_1}{\lambda_1}\\ &= -\frac{\partial E_1}{\partial T_0}\frac{-\frac{\partial r_1^M}{\partial K_1^M}(\lambda_1-1) - \frac{\partial r_1^I}{\partial K_1^I}\lambda_1}{-\frac{\partial r_1^M}{\partial K_1^M}(\lambda_1-1) - \frac{\partial r_1^I}{\partial K_1^I}\lambda_1 + \frac{r_1^M - r_1^I + \theta}{E_1}} > -\frac{\partial E_1}{\partial T_0}\end{aligned}$$

where $\frac{\partial \lambda_1}{\partial E_1}$ is derived from total differentiation of (11).

As aggregate resources increase (the right-hand side of the period 1 resource constraint) in $T_0$, there are also more resources for consumption and investment purposes. The consumption and saving policies now read

$$\begin{aligned}C_1^W &= \frac{1}{1+\tau_1^W}\Big(w_1^M L_1^M + w_1^I L_1^I\Big)\\ C_1^H &= \frac{1-\beta_H}{1+\tau_1^W}(1 + r_1^M - \delta)\Omega_1\\ C_1^B &= \frac{1-\beta_B}{1+\tau_1^B}(\theta\lambda_1 - \delta)E_1\\ \Omega_2 &= \frac{\beta_H}{1+\sigma_1^W}(1 + r_1^M - \delta)\Omega_1\\ E_2 &= \frac{\beta_B}{1+\sigma_1^B}(\theta\lambda_1 - \delta)E_1.\end{aligned}$$

As previously, there are sufficiently many independent policy instruments to engineer a consumption and saving allocation that satisfies $\mathrm{d}C_1^B + \mathrm{d}C_1^H + \mathrm{d}C_1^E + \mathrm{d}E_2 + \mathrm{d}\Omega_2 = dY_1 > 0$. Specifically, we focus on policies such that $\mathrm{d}C_t^B = 0$, $\mathrm{d}C_t^H \geq 0$, $\mathrm{d}C_t^W \geq 0$, $\mathrm{d}E_2 > 0$, $\mathrm{d}\Omega_2 \geq 0$, i.e. policies that lead to a higher equity endowment, which attenuates the propagation of shocks through the bank balance sheet. Note that these policies implement a consumption path of bankers similar to their *laissez-faire* one in order to

[21]Note that labor is supplied inelastically in each sector, and thus only capital reallocation effects matter.

avoid excessive risk taking of banks in the expectation of bailout policies. Applying similar arguments for the subsequent period establishes the proposition.[22] □

Note that in the context of linear homogenous consumption and saving policies, the set of consumption and saving taxes is easy to implement, but there are also different policies that get to the same result. In particular, we emphasize that the effect of the policy $\{\tau_t^B, \sigma_t^B\}$ on the consumption of bankers and their accumulation of equity capital is isomorph to a policy that implements dividend payout restrictions and accordingly adjusted investment subsidies $\{\delta_t^B, \tilde{\sigma}_t^B\}$.

## 7.2 Managing Recoveries with Limited Set of Policy Instruments

Engineering a Pareto optimal recovery may require quite a large set of policy instruments. In this subsection, we examine how to manage recoveries when only two standard policy instruments are available: dividend payout restrictions and capital injections into the banking system financed by taxing investors. Both instruments have been used extensively in the aftermath of the financial crisis 2007 to 2009. In order to simplify the exposition, we abstract from capital depreciation and any further taxes and subsidies for the analytical results.

We investigate how these two policy instruments can be used simultaneously to speed up the recovery and to distribute the gains across agents. Suppose that bank equity declines from by $\Delta E > 0$ at the end of period 0, i.e after the production stage and before savings-investment decisions are made. We consider dividend payout ratios and a capital transfer $\Delta T$ ($\Delta T > 0$) from investors to the banking sector (see our previous discussion). We refer to this policy package as *balanced bailout*.

We construct the policy scheme as follows. Suppose that the regulator makes a small one-time transfer $\Delta T$ from investors to bankers and imposes a payout restriction which forces bankers to retain a fraction $d_0 > \beta_B$ of their wealth. Hence, dividend payouts are restricted to $1 - d_0 < 1 - \beta_B$ at the end of period $\tau = 0$. Then, the regulator follows a scheme that produces the same consumption path for bankers as in the case when there are no policy interventions.[23] A formal description and analysis of the scheme is deferred

[22] Through the construction of the policy there are no incentives for bankers to consume more in particular periods than it would be optimal under the plan characterized in Proposition 6, thereby decreasing bank equity in order to benefit from a bailout is not in the interest of individual bankers. Also collectively bankers could not gain by depleting bank equity through excessive consumption in anticipation of a subsequent bailout.

[23] Policies that would make bankers better off in the accelerated recovery than without policy interventions may introduce moral hazard. Bankers may have an incentive to pay out more dividends, thereby consuming more, in order to cause a negative bank equity shock and a bailout. However, there are no incentives for such a behavior at the individual level as banks can trigger a bailout only collectively.

to Appendix A.5.

Because bankers obtain the same consumption path as under *laissez-faire*, bankers are indifferent between regulation and *laissez-faire*. With balanced bailout the total capital stock, $K_t$, and capital employed in sector $I$, $K_t^I$, exceed their laissez-faire values in all periods. As a result, total wage income of workers is always higher than under *laissez-faire* that workers unambiguously benefit from such a scheme. For investors, however, the result is ambiguous: although benefiting from faster recoveries with higher returns, investors suffer from financing the initial capital injection to the banking sector. Which effect finally dominates depends on the specific calibration. The reason for this is as follows. If leverage is high, bank equity shocks also lead to high reductions in loan supply and thus to high output losses. Recessions are deep and persistent such that policies that avoid the bank capital transmission channel to unfold – like the investor financed capital injections – may be even welfare improving for investors.

We summarize the above results in the following proposition:

**Proposition 7** (Dividend Payout Restrictions and Capital Injections)**.**
*Suppose that a negative bank equity shock occurs and suppose that investors are sufficiently patient. Then, there exists an initial transfer $\Delta T$ from investors to bankers and a sequence of dividend payout restrictions on banks $\{1 - d_t\}_{t=0}^{\infty}$ such that life-time utility of bankers is the same as under laissez-faire and workers are better off. However, the impact on investors is ambiguous.*

*Proof.* See Appendix A.5. □

To sum up, with only two policy instruments, recovery can be sped up, which benefits workers and leaves life-time utility of bankers unchanged. While workers benefit, the welfare consequences for investors is ambiguous and depends on the specific parameterization of the model.

# 8 Quantitative Analysis

This section provides a quantitative assessment of the theoretical results that have been derived in Sections 4 to 7. Specifically, we quantify the impact of shocks to capital, productivity, and the degree of financial friction using a calibrated version of the model that is consistent with stylized facts of the US economy.

### 8.1 Calibration

There are ten model parameters: the production parameters $\alpha, A, z^M$, and $z^I$, the depreciation rate $\delta$, the time preference factors $\beta_H$ and $\beta_B$, the intensity of financial frictions $\theta$, and the labor endowments $L^M$ and $L^I$. We calibrate the steady state of the model on a quarterly base using quarterly and annual US data from 1998 to 2004 from the Federal Reserve Economic Data (FRED), the Federal Deposit Insurance Corporation (FDIC) Call Report Data, and the Penn World Table. Although it is possible to calibrate each parameter sequentially, we relegate the details of the calibration strategy to Appendix B.

There are three normalizations. First, we normalize common factor productivity and choose $A = 1$. Second, as only the relative sectoral productivity $z$ enters the steady state conditions, we normalize productivity in sector $M$ and set $z^M = 1$. Third, as only the relative size of labor endowments $\ell$ enters the steady state conditions, we normalize labor input in sector $M$ and set $L^M = 1$.

The remaining parameters match calibration targets consistent with US time series data. The targets are as follows: First, the output elasticity of capital is $\alpha = 0.3600$, which is in the range of values typically used for the US in the RBC literature. Second, the saving rate is set to $\overline{s} = 0.1872$, which is the average gross-saving-to-GNP ratio in the FRED NIPA accounts. Third, the target for the capital-to-output ratio is based on the Penn-World Table and we set $\overline{K/Y} = 12$ for the quarterly calibration. Fourth, the calibration target for bank leverage $\overline{\lambda} = 10.7449$ is taken from the aggregated Call Report Data provided by the FDIC. Fifth, based on annual data in FRED, we set the quarterly average return on bank equity to $\overline{r}^B = 0.0339$. Sixth, we target a relative size of the banking sector by setting the average bond-to-loan finance ratio to $\overline{K^M/K^I} = 1.5000$ – a value consistent with De Fiore and Uhlig (2011). Seventh, we assume that the relative return differences between sectors only stem from productivity differences and thus capital intensities in both sectors are aligned. Table 2 summarizes the calibrated parameter values and the calibration targets.

In order to assess the calibration strategy, we compute the implied size of the banking sector relative to GDP. Due to the fact that in our model loan supply $K^I$ corresponds to the total asset side of the bank balance sheet, the relative size of the banking sector is $\hat{K}^I/\hat{Y} = (1 + \overline{K^M/K^I})^{-1}\overline{K/Y} = 4.8000$, i.e. 1.2000 on an annual base. However, choosing 1998-2004 as reference period, the relative size of the bank sector in the data is only 0.7839 on an annual base. One reason for the higher relative bank sector size in our model is that we abstract from retained earnings in the production sector as an additional source to finance investment. The steady state allocation and further non-targeted equilibrium statistics are summarized in Table 3.

Table 2: Parameters and Calibration Targets

| | | | | | |
|---|---|---|---|---|---|
| parameter | $\alpha$ | $A$ | $z^M$ | $z^I$ | $\delta$ |
| value (quarterly base) | 0.3600 | 1.0000 | 1.0000 | 1.0644 | 0.0156 |
| parameter | $\beta^H$ | $\beta^B$ | $\theta$ | $L^M$ | $L^I$ |
| value (quarterly base) | 0.9865 | 0.9672 | 0.0977 | 1.0000 | 0.6667 |
| calibration target | $\overline{s}$ | $\overline{K/Y}$ | $\overline{\lambda}$ | $\overline{r}^B$ | $\overline{K^M/K^I}$ |
| value (quarterly base) | 0.1872 | 12.0000 | 10.7449 | 0.0339 | 1.5000 |

Table 3: Steady State Allocation and Non-Targeted Statistics

| | | | | | | |
|---|---|---|---|---|---|---|
| steady state allocation | $\hat{E}$ | $\hat{K}$ | $\hat{K}^M$ | $\hat{K}^I$ | $\hat{C}^H$ | $\hat{C}^B$ |
| value (quarterly base) | 3.1347 | 84.2038 | 50.5223 | 33.6815 | 1.1063 | 0.1062 |
| non-targeted statistic | $\hat{r}^M$ | $\hat{r}^I$ | $\hat{w}^M$ | $\hat{w}^I$ | $\hat{K}^I/\hat{Y}$ | |
| value (quarterly base) | 0.0292 | 0.0311 | 2.6268 | 2.7960 | 4.8000 | |

## 8.2 Wealth Shocks

We consider two different wealth shocks that fall primarily either on bankers or on investors. The main insights of the quantitative analysis are as follows: First, when levered banks provide loans to sectors that would otherwise not have access to capital markets, wealth shocks are amplified and are more persistent. Second, shocks hitting the banking sector directly are initially more severe than shocks hitting investors first, even when shocks are of similar magnitude. The persistence of these shocks, however, is similar.

Shocks are parameterized such that they reduce either bank equity or investor wealth by

1 percent of the total steady state capital stock, i.e. $\Delta E = 0.01\hat{K}$ or $\Delta\Omega = 0.01\hat{K}$.[24] As a result, bank equity and investor wealth shocks amount to 26.86 and 1.04 percent of equity and investor wealth, respectively, and could be associated with a banking crisis and a (moderate) financial crisis, respectively. The size of the bank equity shock is large but not unreasonable for periods like the Great Recession in which inside and outside equity, measured in book or market values, dropped substantially. The impulse response functions of capital, output, investment, leverage, and consumption are depicted in Figure 4.

In order to measure dynamic output losses, we compute the elasticity of the present value of output deviations with respect to capital shocks

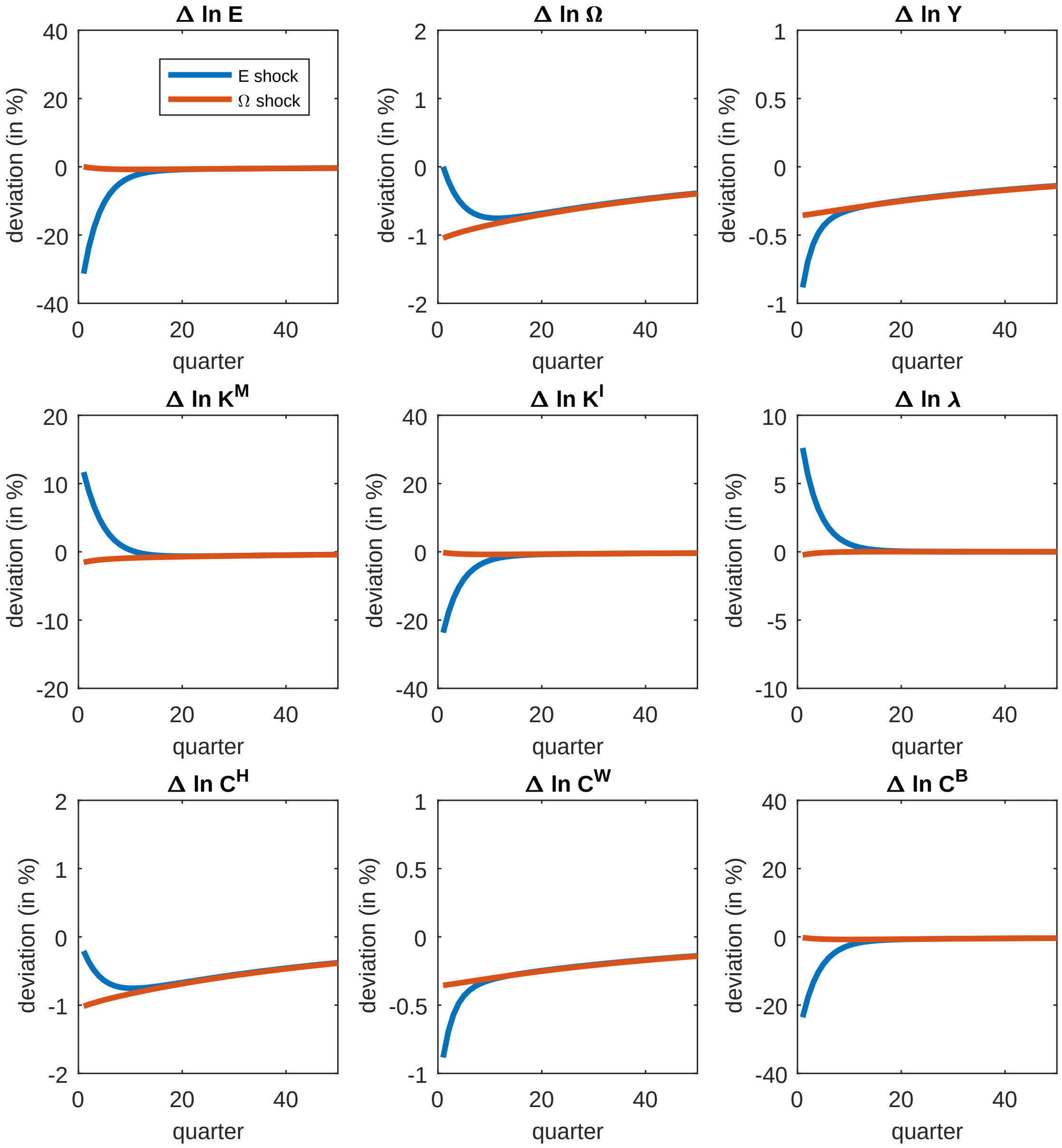
$$\mu \mid_{\Delta K} = \frac{1}{4} \frac{\sum_{t=0}^{\infty} \beta_H^t (Y_t - \hat{Y})}{\hat{Y}} \frac{\hat{K}}{\Delta K}$$

where the adjustment factor $\frac{1}{4}$ adjusts the quarterly calibration to annually interpretable measures. In essence, this measure is a multiplier that reflects the total long-run impact of capital shocks on the accumulated output deviations. Specifically, we define the *bank equity multiplier* $\mu \mid_{\Delta K = \Delta E}$ and the *household wealth multiplier* $\mu \mid_{\Delta K = \Delta\Omega}$ when shocks exclusively fall on banks and investors, respectively. The quantitative results for the *bank equity multiplier* and the *household wealth multiplier* are reported in Table 4. The impact of the bank equity shock is larger, although both values are of the same order of magnitude. The reason is that shocks to investor wealth trigger a subsequent decline of bank equity that delays the recovery and makes shocks similarly persistent as bank equity shocks of comparable size.

Focusing on consumption, we find that both shocks affect consumption of investors and workers in a similar way. Based on the complete consumption path, we follow Lucas (1987) in measuring welfare costs and compute the required permanent increase in period consumption (in percentage points) that would fully compensate agents for the shocks in terms of lifetime utility. Thus, welfare costs are essentially denominated in consumption equivalent units. The welfare costs of a bank equity shock and an investor wealth shock are of the same order of magnitude for workers and investors which indicates that the propagation of shocks through bank balance sheets has important welfare implications, even for agents that are not directly affected by the shock in the initial period. Quantitatively, the welfare cost for investors amount to 0.3652 and 0.4159 percent of consumption equivalent units for bank equity and investor wealth shocks, respectively. For workers, the welfare costs are 0.1696 and 0.1513 percent of consumption equivalent units for bank equity and investor wealth shocks, respectively. Costs are relatively small

[24] Note that when $\Delta X > 0$, variable $X$ declines by $\Delta X$.

Figure 4: Response to Wealth Shock




*Simulation results for a negative shock of 1 percent of total capital to either bank equity or investor wealth. Shocks are at the beginning of the period, i.e. before the production stage.*

but not negligible compared to the cost usually reported for the cost of business cycles that rarely exceed 0.1 of consumption equivalent units (see Barlevy (2004)). In contrast, a bank equity shock leads to much larger welfare cost for bankers (3.3494 percent of consumption equivalent units) than a comparable one on investor wealth (0.5338 percent of consumption equivalent units). Table 4 summarizes the quantitative results. In essence, the quantitative results highlight the role of banks in transmitting adverse shocks even when these banks are not directly affected by shocks in first place.

Table 4: Welfare Cost and Output Multipliers of Wealth Shocks

| | multipliers | welfare cost | | |
|---|---|---|---|---|
| | | investor | worker | banker |
| bank equity shock | 5.0759 | 0.3652 | 0.1696 | 3.3494 |
| investor wealth shock | 4.7639 | 0.4159 | 0.1513 | 0.5338 |

*Simulation results for a negative shock of 1 percent of total capital to either bank equity or investor wealth. Shocks are at the beginning of the period, i.e. before the production stage. Welfare costs are denominated as the required percentage change of permanent consumption to compensate for the specific shocks.*

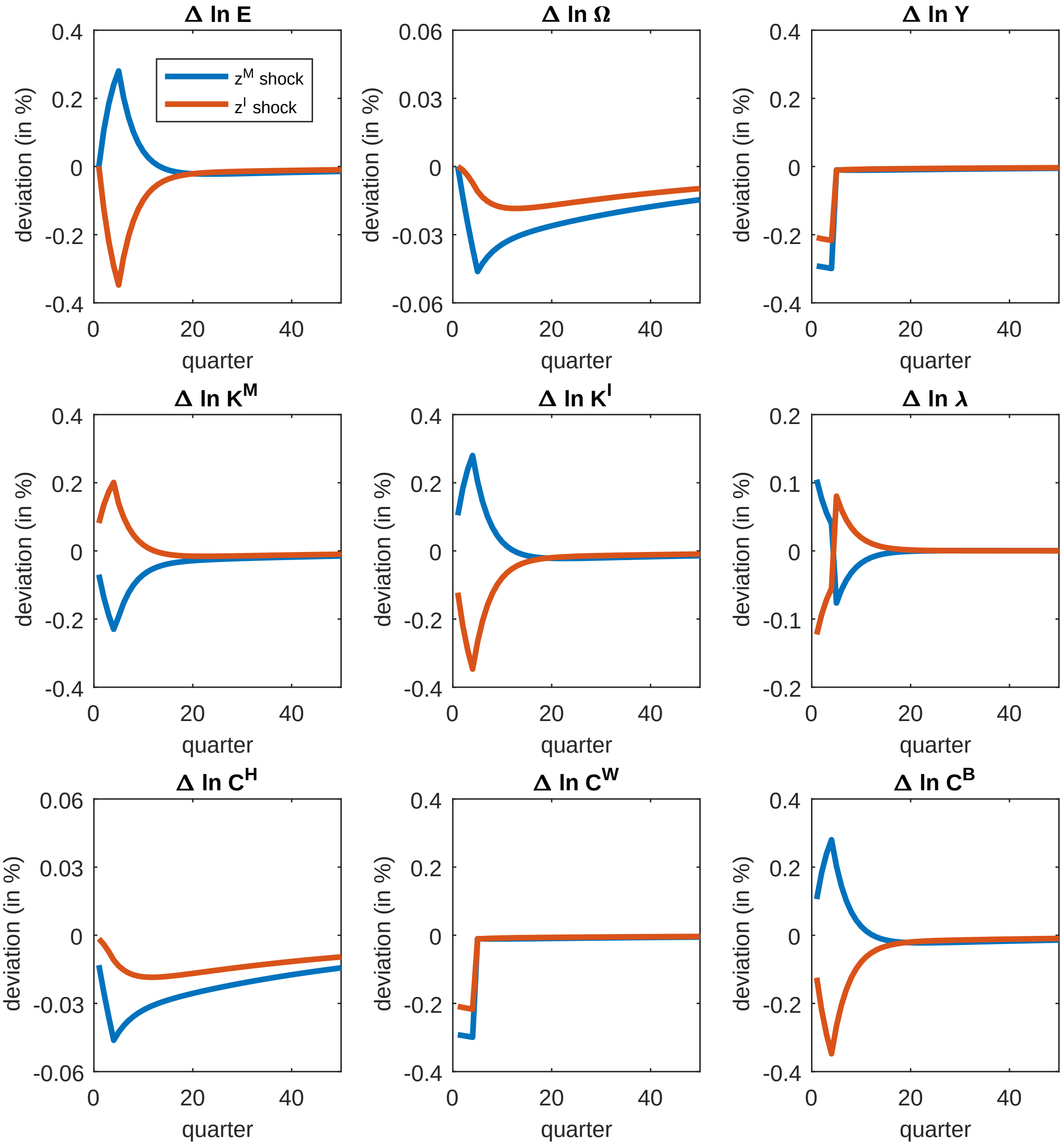
### 8.3 Transitory Productivity Shocks

This section considers isolated productivity shocks in sectors $I$ and $M$ as well as common productivity shocks that leave the relative productivity in both sectors unaffected. The main results are as follows: First, sector-specific shocks lead to substantially different dynamics. While negative shocks in sector $I$ induce a drop in bank equity and investor wealth, shocks in sector $M$ even lead to a temporary boom in the banking sector. Second, as of Corollaries 1 to 3, common productivity shocks induce procyclical bank leverage and countercyclical bond-to-loan finance ratios.

Shocks are parameterized as a 0.5 percent decrease in productivity $z^M$, $z^I$, or $A$ for 4 consecutive quarters. The parametrization is in line with US business cycle statistics. Based on Fernald (2015), we find that on average, total factor productivity has dropped by 0.4 to 0.5 percent during the US recessions since the 1980s. In addition, relying on the *Business Cycle Dating Committee of the National Bureau of Economic Research*, we compute that the average recessions since the 1980s took approximately 11 months. The impulse response functions of capital, output, investment, leverage, and consumption are depicted in Figure 5. As the impulse responses for productivity shocks in sector $I$ are quantitatively and graphically close, we drop the graph of the common productivity shock from Figure 5 for convenience.

Welfare losses in terms of consumption equivalent units are moderate for investors and workers and are typically below 0.02 percent of consumption equivalent units. In con-

Figure 5: Response to Productivity Shock




*Simulation results for a negative shock of 0.5 percent to sector specific productivity or common factor productivity for 4 quarters. Shocks are at the beginning of the period, i.e. before the production stage.*

trast, bankers even realize welfare gains of 0.0310 percent of consumption equivalent units when an isolated productivity shock hits sector $M$. This is due to the fact that the increase in leverage (as of Corollary 1) generates higher profits and therefore raises the bankers' consumption path. In contrast, when the shock hits sector $I$, the bankers' welfare loss amounts to 0.0683 percent of consumption equivalent units and is thus higher than the welfare loss of investors and workers. Finally, for shocks to common productivity, welfare costs amount to 0.0243, 0.0351, and 0.0372 percent of consumption equivalent units for investors, workers, and bankers, respectively. The welfare costs for all shock scenarios are summarized in Table 5.

Table 5: Welfare Cost of Productivity Shocks

| | welfare cost | | |
|---|---|---|---|
| | investor | worker | banker |
| productivity shock in sector $M$ | 0.0155 | 0.0204 | -0.0310 |
| productivity shock in sector $I$ | 0.0088 | 0.0146 | 0.0683 |
| common productivity shock | 0.0243 | 0.0351 | 0.0372 |

*Simulation results for a negative shock of 0.5 percent to sector specific productivity or common factor productivity for 4 quarters. Shocks are at the beginning of the period, i.e. before the production stage. Welfare costs are denominated as the required percentage change of permanent consumption to compensate for the specific shocks.*

### 8.4 Transitory Worsening of Financial Frictions

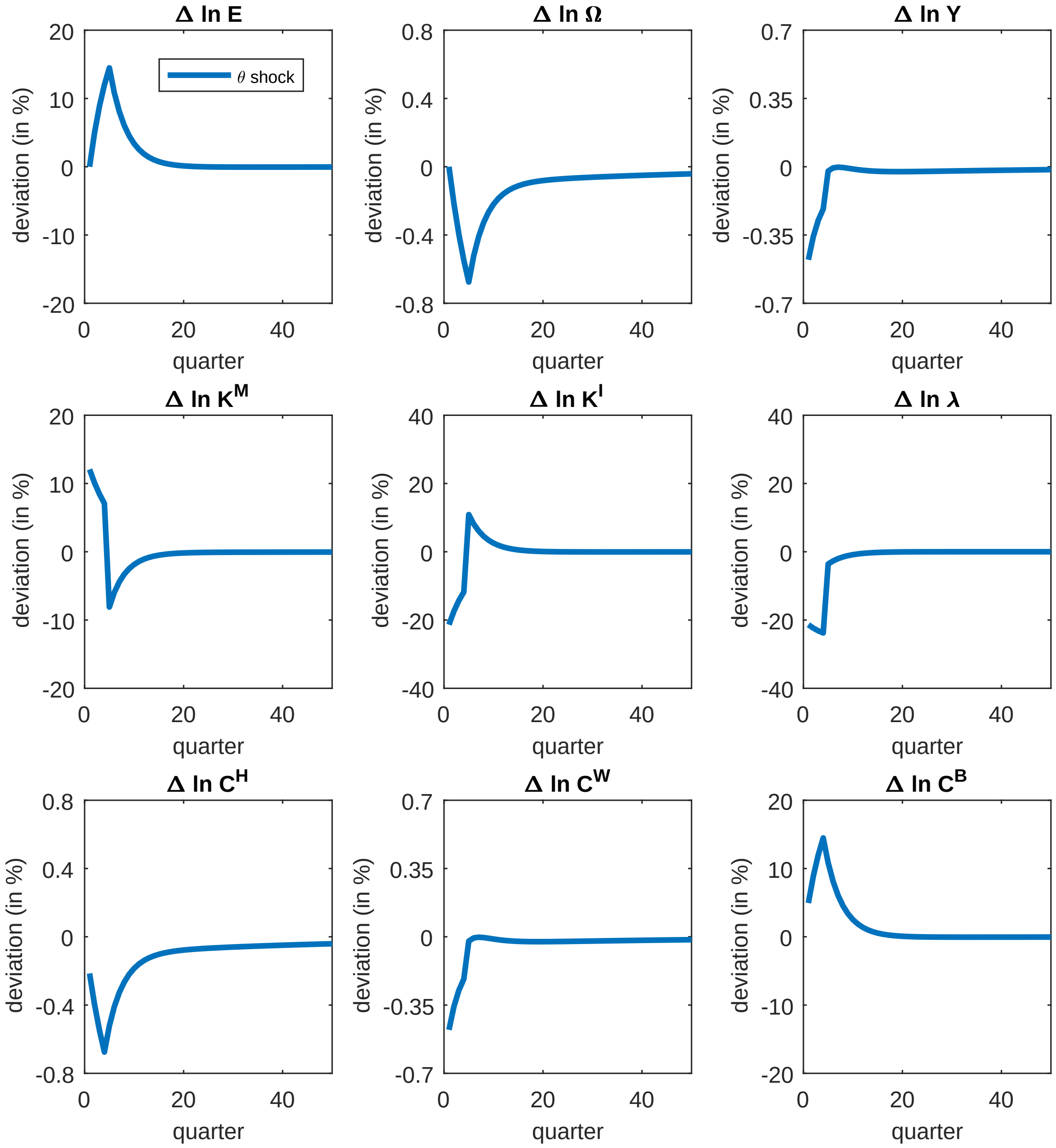
This section discusses the impact of a temporary shock to financial frictions, i.e. $\theta$ increases to $\theta' > \theta$ for some quarters. The main result is as follows: A negative shock to financial frictions boosts accumulation of bank equity while it depresses output. When the financial friction returns to its pre-shock level $\theta$, a higher level of bank equity may allow temporary higher investment in sector $I$, thereby boosting output.

Trust shocks are parameterized as a 30 percent increase in the financial friction for 4 consecutive quarters, being consistent with the average length of the US business cycles since the 1980s.[25] The impulse response functions of capital, output, investment, leverage, and consumption are depicted in Figure 6. Note that for our calibration, output exceeds its steady state level as soon as the measure of the financial friction returns to its pre-shock value. However, the induced boom is quantitatively very small.

Our analysis shows that a worsening of financial frictions leads to comparatively large welfare gains for the bankers of 2.3039 percent of consumption equivalent units through

[25] Zingales (2011) argues that trust dropped dramatically in fall 2008 and suggests orders of magnitude in the range considered in this paper (see also Bloom et al. (2012)).

Figure 6: Response to Trust Shock




*Simulation results for a negative shock of 30 percent to the financial friction $\theta$ for 4 quarters. Shocks are at the beginning of the period, i.e. before the production stage.*

the induced boost in bank equity, whereas investors and workers suffer from a welfare loss of only 0.0828 and 0.0290 percent of consumption equivalent units – see Table 6.

Table 6: Welfare Cost of Trust Shock

| | welfare cost | | |
|---|---|---|---|
| | investor | worker | banker |
| trust shock | 0.0828 | 0.0290 | -2.3039 |

*Simulation results for a negative shock of 30 percent to the financial friction $\theta$ for 4 quarters. Shocks are at the beginning of the period, i.e. before the production stage. Welfare costs are denominated as the required percentage change of permanent consumption to compensate for the specific shocks.*

### 8.5 Bond and Loan Finance over the Business Cycle

The patterns of the two forms of financing – bank loans and bonds – over the business cycle are well-documented: De Fiore and Uhlig (2015) and Contessi et al. (2013) find that total bank loans are markedly procyclical (with some lag), while the level of bonds reacts little or is even countercyclical. We account for this empirical observation in three steps, using the shock parameterizations described in Sections 8.2 to 8.4. Of course, the separation in three steps is solely made for convenience in order to decompose the impact of the tree different kinds of shocks.

Step 1: If normal recessions are associated with a negative shock to common productivity, Corollaries 1 to 3 yield procyclicality of bank leverage and countercyclicality of bond-to-loan finance ratios, i.e. leverage declines and the bond-to-loan ratio increases.

Step 2: If the recession is accompanied by an initial sharp decline in bank equity, we refer to this situation as a *banking crisis*. Following our discussion of bank equity shocks in Section 8.2, we find that the countercyclicality of the bond-to-loan finance ratio get reinforced while procyclicality of bank leverage is dampened (see Figure 4).

Step 3: If the banking crisis is accompanied by an increase of the financial friction $\theta$ as trust in banks decline,[26] the cyclical pattern of bank loans and bonds in downturns of the types described in Step 1 or Step 2, i.e. procyclical bank leverage and countercyclical of the bond-to-loan finance ratio, get reinforced, as discussed in Section 8.4.

[26] According to Bloom et al. (2012), downturns are associated with a general increase of uncertainty, which could be interpreted as less trust in repayment pledges in our context.

Table 7 reports welfare effects associated with the different steps as described above. The main impact on the welfare costs comes from the equity shock that accounts for more than two-third of the welfare costs for investors and workers. Though being also an important driver of the welfare costs for bankers, the change of $\theta$ is a sufficiently strong mechanism that cuts the bank equity shock induced welfare loss of bankers by more than a half.

Table 7: Welfare Cost of Joint Shocks

| | welfare cost | | |
|---|---|---|---|
| | investor | worker | banker |
| productivity shock | 0.0243 | 0.0351 | 0.0372 |
| productivity and equity shock | 0.3895 | 0.2049 | 3.3971 |
| productivity, equity, and trust shock | 0.4689 | 0.2516 | 1.4504 |

*Simulation results for a negative shock of 1 percent of total capital to bank equity, a negative shock of 0.5 percent to common factor productivity, and a negative shock of 30 percent to the financial friction $\theta$ for 4 quarters. Shocks are at the beginning of the period, i.e. before the production stage. Welfare costs are denominated as the required percentage change of permanent consumption to compensate for the specific shocks.*

### 8.6 Speeding Up Recoveries

We now assess the quantitative effects of a *balanced bailout* as defined in Section 7.2. Specifically, suppose there is a shock to bank equity of 1 percent of total capital after the production stage and before consumption and saving decisions are made. The *balanced bailout* implements an initial investor financed capital injection to banks that compensates for 50 percent of the initial bank equity shock and is accompanied by a sequence of dividend payout restrictions that sets the realized consumption path of the bankers to the respective *laissez faire* consumption path – the bankers lifetime utility remains unaffected by the *balanced bailout*.

The welfare cost of bankers for the *balanced bailout* is constant by construction. As discussed in Section 7.2, the welfare cost of workers decreases because the *balanced bailout* accelerates capital accumulation and thus the total wage sum in the economy. In the baseline version of the model with a calibrated steady state value for bank leverage of $\hat{\lambda} = 10.7449$, the welfare cost of investors increases from 0.3482 to 0.3539 percent of consumption equivalent units.[27] This means, that the initial drop in investor wealth and, thus, investor consumption (due to the enforced transfer payment to bankers) dominates the positive effect of an accelerated recovery on future investor wealth and consumption. However, considering a re-calibrated version of the model with a targeted bank leverage of $\hat{\lambda} = 21.4898$ reveals that the *balanced bailout* can be welfare improving even for investors as the welfare cost declines from 0.3894 to 0.3822 percent of consumption equivalent units. The reason for the declining welfare cost is that the larger the initial bank leverage, the larger the decline of loan supply in response of a bank equity shock, and the more likely it is to trigger the persistent and costly *bank lending channel* that delays economic recovery. Thus, even for investors who pay for the initial capital injection, the impact of an accelerated recovery can be sufficiently strong to compensate for the initial consumption and utility loss.

Table 8: Welfare Cost of Equity Shocks under *Balanced Bailout*

| Policy | welfare cost | | |
|---|---|---|---|
| | investor | worker | banker |
| $\hat{\lambda} = 10.74$ | | | |
| *laissez-faire* | 0.3482 | 0.1613 | 4.1383 |
| *balanced bailout* | 0.3539 | 0.1380 | 4.1383 |
| $\hat{\lambda} = 21.48$ | | | |
| *laissez-faire* | 0.3894 | 0.1772 | 6.1883 |
| *balanced bailout* | 0.3822 | 0.1467 | 6.1883 |

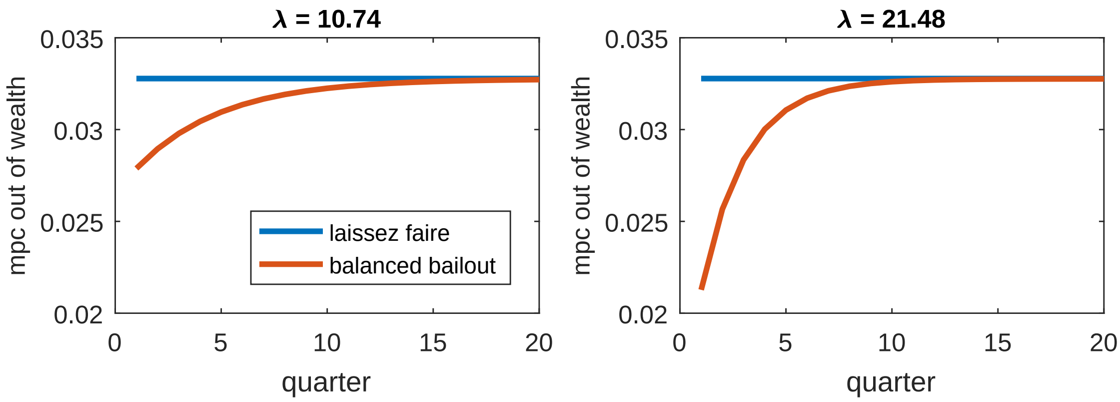
*Simulation results for a negative shock of 1 percent of total capital to bank equity. Shocks are at the end of the period, i.e. after the production stage and before consumption and saving decisions are made. The balanced bailout is computed for an initial transfer payment that compensates for 50 percent of the equity shock. For the second set of results, the model is re-calibrated to a steady state leverage of $\hat{\lambda} = 21.48$. Welfare costs are denominated as the required percentage change of permanent consumption to compensate for the specific shocks.*

Figure 7 compares the bankers marginal propensity to consume out of wealth under the different scenarios (*laissez faire* and *balanced bailout*) and for the different calibration

[27] Note that the numbers in Table 8 differ from the numbers in Table 4 because the sequence of shocks differs: in Table 8 shocks are after the production stage and in Table 4 shocks are before the production stage.

targets. As can be seen, the dividend payout restrictions that materializes in the gap between (*laissez faire* and *balanced bailout*) marginal propensities to consume out of wealth are substantial and take more than three years to vanish.

Figure 7: Dividend Payout Restrictions under *Balanced Bailout*




*Simulation results for a negative shock of 1 percent of total capital to bank equity. Shocks are at the end of the period, i.e. after the production stage and before consumption and saving decisions are made. The balanced bailout is computed for an initial transfer payment that compensates for 50 percent of the equity shock. For the second set of results, the model is re-calibrated to a steady state leverage of* $\hat{\lambda} = 21.48$.

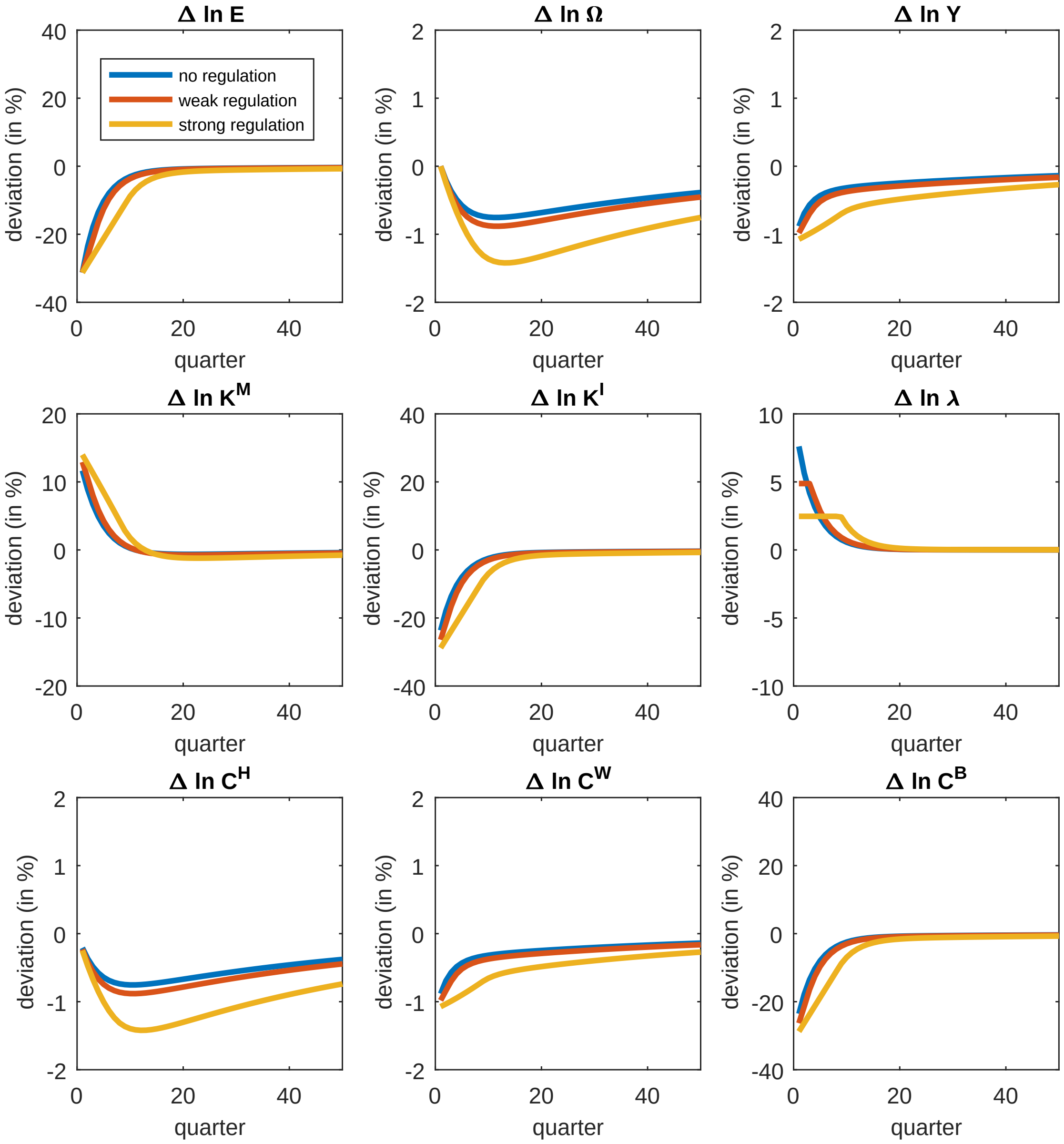
## 8.7 Capital Regulation and Automatic Stabilization

We now evaluate the impact of capital regulation that partially shuts down the automatic stabilization of bank leverage as discussed in Section 6.2. Suppose that there is a shock to bank equity of 1 percent of total capital. Consider three situations: first, *laissez faire*, second, a leverage limit of 5.0 percent above steady state leverage (weak regulations), and, third, a leverage limit of 2.5 percent above steady state leverage (strong regulations).

Figure 8 shows that the stronger the regulation, i.e. the weaker the automatic stabilization, the more persistent is the bank equity shock. There is an intuitive explanation for this result if we consider an *extreme* form of regulation where leverage is not allowed to exceed its steady state level. In this case, $E_{t+1} = E_t$, such that equity will never rebound and the shock leads to permanent decline in production.

Table 9 quantifies the impact on regulation. Even small differences in the regulatory bank leverage may lead to strong impact on welfare. Specifically, the bank leverage limit under the strong regulation is only by 0.1686 units lower than under weak regulation, which corresponds to a capital ratio differential of around 0.1 percentage points. Nevertheless, welfare costs for investors, workers, and bankers, increase by approximately one third.

Figure 8: Response to Equity Shock under Capital Regulation




*Simulation results for a negative shock of 1 percent of total capital to bank equity. Shocks are at the beginning of the period, i.e. before the production stage. The capital regulation for the weak and strong regulation scenario imposes an upper limit of leverage at 5 percent and 2.5 percent above steady state leverage, respectively.*

## 9 Summary, Extensions, and Outlook

We have presented a simple model of capital accumulation in which financial intermediaries are essential for small and medium firms to invest. The model delivers a set of

Table 9: Welfare Cost and Output Multipliers of Equity Shock under Capital Regulation

| | multipliers | welfare cost | | |
|---|---|---|---|---|
| | | investor | worker | banker |
| no regulation | 5.0759 | 0.3652 | 0.1696 | 3.3494 |
| weak regulation | 5.9630 | 0.4272 | 0.1996 | 3.9686 |
| strong regulation | 9.7380 | 0.6868 | 0.3225 | 6.4164 |

*Simulation results for a negative shock of 1 percent of total capital to bank equity. Shocks are at the beginning of the period, i.e. before the production stage. The capital regulation for the weak and strong regulation scenario imposes an upper limit of leverage at 5 percent and 2.5 percent above steady state leverage, respectively. Welfare costs are denominated as the required percentage change of permanent consumption to compensate for the specific shocks.*

insights into the underlying shock propagation mechanism, is consistent with various stylized facts, and allows to study policy responses to downturns associated with a decline of bank equity. The model presented in this paper is analytically tractable and it can be extended in many ways and, thus, is a convenient module that can be embedded in more complex models. Appendix C outlines four simple extensions that shed further light on the forces at work, the robustness, and interpretation of our findings. In particular, we discuss costs of financial intermediation, a variation with households acting as investors and workers, anticipated bank equity shocks, and stochastic productivity shocks. While these extensions produce essentially the same steady state properties as in the version of the main body of this paper, the transitional dynamics are more complex and – for some extensions – analytically not tractable.

There are numerous further generalizations and extensions. Here, we limit ourselves to briefly outlining three promising avenues for further research with our framework. First, as the Eurozone and a great part of Asia rely heavily on bank loans, while corporate bonds are much more dominant in the US,[28] our framework can help to investigate which type of economic structure is more resilient to adverse shocks. Second, apart from monitoring firms, banks also perform risk sharing and maturity transformation. Including these functions into our banking model with capital accumulation is challenging but can provide further valuable insights. Third, introducing frictional labor markets with imperfect labor transition between production sectors can shed light on how labor market and financial frictions jointly affect amplification and persistence of adverse shocks.

[28] See e.g. De Fiore and Uhlig (2011) and Ghosh (2006).

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# A Appendix – Proofs

## A.1 Appendix – Proof of Corollary 1

The partial derivatives of $\varphi(\lambda_t)$ read

$$
\begin{aligned}
\frac{\partial \varphi(\lambda_t)}{\partial \lambda_t} &= -\frac{\partial r_t^M(K_t^M)}{\partial K_t^M}(\lambda_t - 1)E_t - \frac{\partial r_t^I(K_t^I)}{\partial K_t^I}\lambda_t E_t + r_t^M(K_t^M) - r_t^I(K_t^I) + \theta > 0 \\
\frac{\partial \varphi(\lambda_t)}{\partial \Omega_t} &= \frac{\partial r_t^M(K_t^M)}{\partial K_t^M}(\lambda_t - 1) < 0 \\
\frac{\partial \varphi(\lambda_t)}{\partial E_t} &= -\frac{\partial r_t^M(K_t^M)}{\partial K_t^M}(\lambda_t - 1)^2 - \frac{\partial r_t^I(K_t^I)}{\partial K_t^I}\lambda_t^2 > 0 \\
\frac{\partial \varphi(\lambda_t)}{\partial z^M} &= \frac{r_t^M(K_t^M)}{z^M}(\lambda_t - 1) > 0 \\
\frac{\partial \varphi(\lambda_t)}{\partial z^I} &= -\frac{r_t^I(K_t^I)}{z^I}\lambda_t < 0 \\
\frac{\partial \varphi(\lambda_t)}{\partial A_t} &= \frac{(r_t^M(K_t^M) - r_t^I(K_t^I))\lambda_t - r_t^M(K_t^M)}{A_t} < 0 \\
\frac{\partial \varphi(\lambda_t)}{\partial \theta} &= \lambda_t > 0.
\end{aligned}
$$

Because of inelastic labor supply and the Inada conditions, there will always be production in both sectors, such that $K_t^I > 0$. Therefore, the leverage constraint implies $r_t^M(K_t^M) - r_t^I(K_t^I) + \theta > 0$. The inequalities follow from $r_t^M(K_t^M) - r_t^I(K_t^I) + \theta > 0$, $\lambda_t > 1$, and $\frac{\partial r_t^j(K_t^j)}{\partial K_t^j} < 0$. Total differentiation of $\varphi(\lambda_t)$ and applying the implicit function theorem yield $\frac{\partial \lambda_t}{\partial A_t} > 0$, $\frac{\partial \lambda_t}{\partial z^I} > 0$, $\frac{\partial \lambda_t}{\partial z^M} < 0$, $\frac{\partial \lambda_t}{\partial \Omega_t} > 0$, $\frac{\partial \lambda_t}{\partial E_t} < 0$, and $\frac{\partial \lambda_t}{\partial \theta} < 0$.

## A.2 Appendix – Proof of Corollary 2

First, consider the impact on loan finance, $K_t^I$. When financial frictions are relevant, we have $K_t^I = \lambda_t E_t$. As of Corollary 1, $\frac{\partial K_t^I}{\partial A_t} = \frac{\partial \lambda_t}{\partial A_t}E_t > 0$ and $\frac{\partial K_t^I}{\partial \Omega_t} = \frac{\partial \lambda_t}{\partial \Omega_t}E_t > 0$. Moreover, using Equation (11) yields

$$
\begin{aligned}
\frac{\partial K_t^I}{\partial E_t} &= \frac{\partial \lambda_t}{\partial E_t}E_t + \lambda_t \\
&= -\frac{-\frac{\partial r_t^M(K_t^M)}{\partial K_t^M}(\lambda_t - 1)^2 E_t - \frac{\partial r_t^I(K_t^I)}{\partial K_t^I \lambda_t^2 E_t}}{-\frac{\partial r_t^M(K_t^M)}{\partial K_t^M}(\lambda_t - 1)E_t - \frac{\partial r_t^I(K_t^I)}{\partial K_t^I}\lambda_t E_t + r_t^M(K_t^M) - r_t^I(K_t^I) + \theta} + \lambda_t \\
&= \frac{\frac{-\partial r_t^M(K_t^M)}{\partial K_t^M}(\lambda_t - 1)E_t + \lambda_t(r_t^M(K_t^M) - r_t^I(K_t^I) + \theta)}{-\frac{\partial r_t^M(K_t^M)}{\partial K_t^M}(\lambda_t - 1)E_t - \frac{\partial r_t^I(K_t^I)}{\partial K_t^I}\lambda_t E_t + r_t^M(K_t^M) - r_t^I(K_t^I) + \theta} > 0.
\end{aligned}
$$

The inequality follows from $r_t^M(K_t^M) - r_t^I(K_t^I) + \theta > 0$, $\lambda_t > 1$, and $\frac{\partial r_t^j(K_t^j)}{\partial K_t^j} < 0$.

Second, consider the impact on bond finance, $K_t^M$. When financial frictions are relevant, $K_t^M = E_t + \Omega_t - \lambda_t E_t$. As of Corollary 1 $\frac{\partial K_t^M}{\partial A_t} = -\frac{\partial \lambda_t}{\partial A_t} E_t < 0$. Moreover,

$$
\begin{aligned}
\frac{\partial K_t^M}{\partial \Omega_t} &= 1 - \frac{\partial \lambda_t}{\partial \Omega_t} E_t \\
&= 1 + \frac{\frac{\partial r_t^M(K_t^M)}{\partial K_t^M}(\lambda_t - 1)E_t}{-\frac{\partial r_t^M(K_t^M)}{\partial K_t^M}(\lambda_t - 1)E_t - \frac{\partial r_t^I(K_t^I)}{\partial K_t^I}\lambda_t E_t + r_t^M(K_t^M) - r_t^I(K_t^I) + \theta} \\
&= \frac{-\frac{\partial r_t^I(K_t^I)}{\partial K_t^I}\lambda_t E_t + r_t^M(K_t^M) - r_t^I(K_t^I) + \theta}{-\frac{\partial r_t^M(K_t^M)}{\partial K_t^M}(\lambda_t - 1)E_t - \frac{\partial r_t^I(K_t^I)}{\partial K_t^I}\lambda_t E_t + r_t^M(K_t^M) - r_t^I(K_t^I) + \theta} > 0 \\
\frac{\partial K_t^M}{\partial E_t} &= 1 - \left(\frac{\partial \lambda_t}{\partial E_t} E_t + \lambda_t\right) \\
&= 1 + \frac{-\frac{\partial r_t^M(K_t^M)}{\partial K_t^M}(\lambda_t - 1)^2 E_t - \frac{\partial r_t^I(K_t^I)}{\partial K_t^I}\lambda_t^2 E_t}{-\frac{\partial r_t^M(K_t^M)}{\partial K_t^M}(\lambda_t - 1)E_t - \frac{\partial r_t^I(K_t^I)}{\partial K_t^I}\lambda_t E_t + r_t^M(K_t^M) - r_t^I(K_t^I) + \theta} - \lambda_t \\
&= -\frac{\frac{\partial r_t^I(K_t^I)}{\partial K_t^I}\lambda_t E_t + (\lambda_t - 1)(r_t^M(K_t^M) - r_t^I(K_t^I) + \theta)}{-\frac{\partial r_t^M(K_t^M)}{\partial K_t^M}(\lambda_t - 1)E_t - \frac{\partial r_t^I}{\partial K_t^I}\lambda_t E_t + r_t^M(K_t^M) - r_t^I(K_t^I) + \theta} \\
&= -\frac{1 + \frac{\partial r_t^I(K_t^I)}{\partial K_t^I}K_t^I + r_t^I(K_t^I) - \theta}{-\frac{\partial r_t^M(K_t^M)}{\partial K_t^M}(\lambda_t - 1)E_t - \frac{\partial r_t^I(K_t^I)}{\partial K_t^I}\lambda_t E_t + r_t^M(K_t^M) - r_t^I(K_t^I) + \theta} < 0,
\end{aligned}
$$

where the third line uses the definition of leverage to rewrite $(\lambda_t - 1)(r_t^M(K_t^M) - r_t^I(K_t^I) + \theta)$ as $1 + r_t^I(K_t^I) - \theta$. The inequalities follow from $r_t^M(K_t^M) - r_t^I(K_t^I) + \theta > 0$, $\lambda_t > 1$, $\frac{\partial r_t^j(K_t^j)}{\partial K_t^j} < 0$, and $\frac{\partial r_t^I(K_t^I)}{\partial K_t^I}K_t^I + r_t^I(K_t^I) = \alpha r_t^I(K_t^I) > 0$.

The proof of point (iii) is obvious.

## A.3 Appendix – Proof of Corollary 3

Note that $Y_t = Y_t^M + Y_t^I$. Thus, $\frac{\partial Y_t}{\partial A_t} = \frac{\partial Y_t^M}{\partial A_t} + \frac{\partial Y_t^I}{\partial A_t} > 0$. When financial frictions are binding, $\frac{\partial K_t^M}{\partial \Omega_t} = 1 - \frac{\partial K_t^I}{\partial \Omega_t}$ and $\frac{\partial K_t^M}{\partial E_t} = 1 - \frac{\partial K_t^I}{\partial E_t}$. Thus, using Equation (11) yields

$$
\begin{aligned}
\frac{\partial Y_t}{\partial \Omega_t} &= r_t^M(K_t^M)\frac{\partial K_t^M}{\partial \Omega_t} + r_t^I(K_t^I)\frac{\partial K_t^I}{\partial \Omega_t} = r_t^M(K_t^M) + (r_t^I(K_t^I) - r_t^M(K_t^M))\frac{\partial K_t^I}{\partial \Omega_t} > 0 \\
\frac{\partial Y_t}{\partial E_t} &= r_t^M(K_t^M)\frac{\partial K_t^M}{\partial E_t} + r_t^I(K_t^I)\frac{\partial K_t^I}{\partial E_t} = r_t^M(K_t^M) + (r_t^I(K_t^I) - r_t^M(K_t^M))\frac{\partial K_t^I}{\partial E_t} > 0.
\end{aligned}
$$

The inequalities follow from $\frac{\partial K_t^I}{\partial \Omega_t} > 0$, $\frac{\partial K_t^I}{\partial E_t} > 0$, and $r_t^I > r_t^M$.

### A.4 Appendix – Proof of Proposition 2

We consider a general structure that applies to both, the banks' and the investors' optimization problem. Consider the following optimization problem

$$\max_{\{C_t, A_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \ln C_t$$

subject to

$$(1+\tau_t)C_t + (1+\sigma_t)A_{t+1} = R_t A_t$$

where $C_t$ is consumption, $A_t$ is the agent's net worth, $\tau_t$ is a consumption tax rate, $\sigma_t$ is an investment subsidy,[29] and $\beta$ is the discount factor. The problem captures of course also the cases $\tau_t = 0$ and $\sigma_t = 0$. The Euler equation reads

$$\frac{1+\sigma_t}{1+\tau_t}\frac{1}{C_t} = \beta \frac{R_{t+1}}{(1+\tau_{t+1})C_{t+1}}.$$

Suppose the consumption policy is $C_t = \frac{\tilde{c}}{1+\tau_t} R_t A_t$ where $\tilde{c}$ is a constant. The Euler equation in conjunction with the budget constraint can be rewritten as follows

$$\begin{aligned}
& \frac{1+\sigma_t}{1+\tau_t}\frac{1}{C_t} = \beta \frac{R_{t+1}}{(1+\tau_{t+1})C_{t+1}} \\
\Leftrightarrow \quad & \frac{1+\sigma_t}{\tilde{c} R_t A_t} = \beta \frac{R_{t+1}}{\tilde{c} R_{t+1} A_{t+1}} \\
\Leftrightarrow \quad & \frac{1+\sigma_t}{\tilde{c} R_t A_t} = \beta \frac{1}{\tilde{c} A_{t+1}} \\
\Leftrightarrow \quad & \frac{1+\sigma_t}{\tilde{c} R_t A_t} = \beta \frac{1+\sigma_t}{\tilde{c}(R_t A_t - (1+\tau_t)C_t)} \\
\Leftrightarrow \quad & \frac{1}{R_t A_t} = \beta \frac{1}{R_t A_t - (1+\tau_t)C_t} \\
\Leftrightarrow \quad & \frac{1}{R_t A_t} = \beta \frac{1}{R_t A_t - \tilde{c} R_t A_t} \\
\Leftrightarrow \quad & \frac{1}{R_t A_t} = \beta \frac{1}{(1-\tilde{c}) R_t A_t} \\
\Leftrightarrow \quad & \tilde{c} = 1 - \beta.
\end{aligned}$$

[29] An investment subsidy benefits investors for their total investment into both sectors by a factor $1+\sigma_t$.

Thus, $C_t = \frac{1-\beta}{1+\tau_t} R_t A_t$. Moreover, the budget constraint yields

$$\begin{aligned}
A_{t+1} &= \frac{1}{1+\sigma_t}(R_t A_t - (1+\tau_t)C_t) \\
&= \frac{1}{1+\sigma_t}(R_t A_t - (1-\beta)R_t A_t) \\
&= \frac{1}{1+\sigma_t}\beta R_t A_t.
\end{aligned}$$

### A.5 Appendix – Proof of Proposition 7

We abstract from capital depreciation and any further taxes and subsidies. Consumption of bankers in period $t = 0$ is constant if

$$\begin{aligned}
&(1-d_0)(\hat{R}_0^B \hat{E} - \Delta E + \Delta T) = (1-\beta_B)(\hat{R}_0^B \hat{E} - \Delta E) \\
\Leftrightarrow \quad & d_0 = 1 - (1-\beta_B)\frac{\hat{R}_0^B \hat{E} - \Delta E}{\hat{R}_0^B \hat{E} - \Delta E + \Delta T}.
\end{aligned} \tag{A1}$$

where $\hat{R}_0^B = 1 + \hat{r}_0^B$ is the return on equity factor.

We next observe that

$$E_1^{bb} = d_0(\hat{R}_0^B \hat{E} - \Delta E + \Delta T) = \beta_B(\hat{R}_0^B \hat{E} - \Delta E) + \Delta T = E_1^{lf} + \Delta T > E_1^{lf}. \tag{A2}$$

$E_1^{bb}$ and $E_1^{lf}$ denote the bank equity levels with balanced bailout and *laissez-faire*, respectively. Property (A2) is a direct consequence of the scheme enacted in period 0. We next observe that

$$E_1^{bb} + \Omega_1^{bb} > E_1^{lf} + \Omega_1^{lf}. \tag{A3}$$

The reason why inequality (A3) holds is that $E_1^{bb} = E_1^{lf} + \Delta T$ while households suffering a loss of $\Delta T$ will reduce savings by $\beta_H(\Delta T) < \Delta T$.

As a consequence of Corollary 1, $\lambda_1^{bb} < \lambda_1^{lf}$. However, we have

$$\lambda_1^{bb} E_1^{bb} > \lambda_1^{lf} E_1^{lf}. \tag{A4}$$

The latter property follows from the following contradiction. Suppose that $\lambda_1^{bb} E_1^{bb} = K_1^{I,bb} \leq K_1^{I,lf} = \lambda_1^{lf} E_1^{lf}$. Hence, $K_1^{M,bb} \geq K_1^{M,lf}$. As a consequence, $r_1^{I,bb} \geq r_1^{I,lf}$ and $r_1^{M,bb} \leq r_1^{M,lf}$. This leads to $\lambda_1^{bb} \geq \lambda_1^{lf}$ which is a contradiction.

At the end of period 1 before consumption/savings decisions are made, the banker's wealth $\theta \lambda_1^{bb} E_1^{bb}$ with the balanced bailout is higher than under *laissez-faire*. Hence, the policy-maker can again impose a dividend payout restriction $1-d_1 < 1-\beta_B$ that satisfies

$$(1-d_1)\theta\lambda_1^{bb}E_1^{bb} = (1-\beta_B)\theta\lambda_1^{lf}E_1^{lf}.$$

We next show that $r_1^{M,bb} > r_1^{M,lf}$ for $\beta_H$ sufficiently high. Suppose to the contrary that $r_1^{M,bb} \leq r_1^{M,lf}$, i.e.

$$\begin{aligned} & K_1^{I,bb} < K_1^{I,lf} + \Delta T(1-\beta_H) \\ \Leftrightarrow \quad & \frac{1+r_1^{M,bb}}{r_1^{M,bb} - r_1^{I,bb} + \theta}(E^{lf} + \Delta T) \leq \frac{1+r_1^{M,lf}}{r_1^{M,lf} - r_1^{I,lf} + \theta}E^{lf} + \Delta T(1-\beta_H). \end{aligned}$$

Substituting $r_1^{M,lf}$ for $r_1^{M,bb}$ further decreases the left-hand side of the inequality and simplifying the condition then yields

$$\begin{aligned} E_1^{lf}(-r_1^{I,lf}) + \Delta T(1+r_1^{M,lf})(r_1^{M,lf} - r_1^{I,lf} + \theta) < \\ E_1^{lf}(-r_1^{I,bb}) + \Delta T(1-\beta_H)(r_1^{M,lf} - r_1^{I,bb} + \theta). \end{aligned}$$

For $\beta_H$ sufficiently close to 1, the relationship can only hold if $K_1^{I,bb} > K_1^{I,lf} + \Delta T(1-\beta_H)$ which is a contradiction. Hence, $r_1^{M,bb} > r_1^{M,lf}$, $\lambda_1^{bb}E_1^{bb} > \lambda_1^{lf}E_1^{lf}$, and $E_1^{bb} + \Omega_1^{bb} > E_1^{lf} + \Omega_1^{lf}$. We obtain

$$\begin{aligned} \Omega_1^{bb}(1+r_1^{M,bb}) + \theta\lambda_1^{bb}E_1^{bb} = \alpha\left\{ z^M\left(\frac{K_1^{M,bb}}{L^M}\right)^\alpha + z^I\left(\frac{K_1^{I,bb}}{L^I}\right)^\alpha \right\} > \\ \alpha\left\{ z^M\left(\frac{K_1^{M,lf}}{L^M}\right)^\alpha + z^I\left(\frac{K_1^{I,lf}}{L^I}\right)^\alpha \right\} = \Omega_1^{lf}(1+r_1^{M,lf}) + \theta\lambda_1^{lf}E_1^{lf}, \quad \text{(A5)} \end{aligned}$$

as $r_1^{I,bb} > r_1^{M,bb} > r_1^{M,lf}$ and $K_1^{M,bb} + K_1^{I,bb} > K_1^{M,lf} + K_1^{I,lf}$, more capital is invested in the more productive sector.

At the end of period 1, households save $\beta_H(1+r_1^{M,bb})\Omega_1^{bb}$ and bankers are forced to save according to the dividend payout restriction

$$d_1\theta\lambda_1^{bb}E_1^{bb} = \theta\lambda_1^{bb}E_1^{bb} - \theta\lambda_1^{lf}E_1^{lf} + \beta_B\theta\lambda_1^{lf}E_1^{lf}.$$

In combination with (A5), we get

$$\beta_H(1+r_1^{M,bb})\Omega_1^{bb} + d_1\theta\lambda_1^{bb}E_1^{bb} > \beta_H(1+r_1^{M,lf})\Omega_1^{lf} + \beta_B\theta\lambda_1^{lf}E_1^{lf}.$$

Hence, after policy-makers impose a dividend payout restriction at the end of period 1 on bank equity – thereby keeping consumption of bankers unchanged – we obtain

$$E_2^{bb} > E_2^{lf} \quad \text{and} \quad E_2^{bb} + \Omega_2^{bb} > E_2^{lf} + \Omega_2^{lf}.$$

A similar logic applies for the second and all subsequent periods.

# B Appendix – Calibration

There are ten model parameters: the production parameters $\alpha, z^M$, and $z^M$, the depreciation rates $\delta^H$ and $\delta^B$, the time preference factors $\beta_H$ and $\beta_B$, the financial friction $\theta$, and labor endowment $L^M$ and $L^I$. We calibrate the model to quarterly frequency using quarterly and annual US data from 1998 to 2004 taken from the Federal Reserve Economic Data (FRED), the Federal Deposit Insurance Corporation (FDIC) Call Report Data, and the Penn World Table (PWT) and proceed as follows. As only the relative productivity affects the steady state allocation, we normalize $z^M = 1$ without loss of generality. In a similar vein, we normalize $L^M = 1$. The output elasticity of capital is set to $\alpha = 0.36$, which is in the range of values suggested in the literature.

We equalize depreciation rates on household wealth and bank equity, i.e. $\delta^B = \delta^H = \delta$. Let $\overline{s}$ denote the aggregate saving rate. The capital-to-output ratio simplifies to $K/Y = s/\delta$. Choosing the saving rate to match the gross-saving-to-GNP ratio taken from the FRED NIPA accounts, i.e. $\overline{s} = 0.1872$, and setting the capital-to-output ratio equal to the respective series from the Penn World Table, i.e. $\overline{K/Y} = 12$ on a quarterly base, delivers $\delta = 0.0156$.

According to FRED, we set the average annual return on bank equity to 14.26 percent which produces a quarterly return $(1 + \overline{r}^B)^4 = 1.1426$, i.e. $\overline{r}^B = 0.0339$. Moreover, taking the calibration target for bank leverage from the aggregated Call Report Data provided by the FDIC, $\overline{\lambda} = 10.7449$, the definition for return on equity $\overline{r}^B = \theta\overline{\lambda} - \delta - 1$ yields $\theta = 0.0977$ and the steady state condition for bank equity yields $\beta_B = 1/(\theta\overline{\lambda} - \delta) = 0.9672$.

According to De Fiore and Uhlig (2011), the average bond-to-loan finance ratio in the US is $\overline{K^M/K^I} = 1.5000$. In order to attribute return differences solely to relative differences in productivity $z$, capital-to-labor ratios in sector $M$ and $I$ have to be equal. Thus, we set $L^I = 2/3$ as $L_t^M$ has been normalized to 1. Furthermore, combining the target for the bond-to-loan finance ratio with the aggregate resource constraint gives $K^I = K/(1 + \overline{K^M/K^I})$. The steady state condition for investor wealth and the leverage condition with $A = 1$ read

$$\frac{K^M}{L^M} = \left(\frac{\alpha z^M}{\frac{1}{\beta_H} + \delta - 1}\right)^{1/(1-\alpha)}$$
$$\frac{K^I}{L^I} = \left(\frac{\alpha\overline{\lambda} z^I}{(\overline{\lambda} - 1)(\frac{1}{\beta_H} + \delta - 1) + \theta\overline{\lambda} - 1}\right)^{1/(1-\alpha)}.$$

Using $K^M = \overline{K^M/K^I} K^I$, we combine both conditions to substitute for $K^I$ and obtain

$$z^I (L^I)^{1-\alpha} = \frac{(\overline{\lambda} - 1)(\frac{1}{\beta_H} + \delta - 1) + \theta\overline{\lambda} - 1}{\frac{1}{\beta_H} + \delta - 1} \frac{z^M}{\overline{\lambda}} \left(\frac{L^M}{\overline{K^M/K^I}}\right)^{1-\alpha}. \tag{B1}$$

Moreover, the capital-to-output ratio is

$$\begin{aligned} \overline{K/Y} &= \frac{(1 + \overline{K^M/K^I})K^I}{z^I (K^I)^\alpha (L^I)^{1-\alpha} + z^M (\overline{K^M/K^I})^\alpha (K^I)^\alpha (L^M)^{1-\alpha}} \\ &= \frac{1 + \overline{K^M/K^I}}{z^I (L^I)^{1-\alpha} + z^M (\overline{K^M/K^I})^\alpha (L^M)^{1-\alpha}} (\overline{K^M/K^I})^{\alpha-1} \frac{\alpha z^M}{\frac{1}{\beta_H} + \delta - 1} (L^M)^{1-\alpha}, \end{aligned} \tag{B2}$$

where the second line uses the steady state condition for investor wealth and the bond-to-loan finance ratio to substitute for $K^I$. Using (B1) in (B2) and solving for $\beta_H$, we obtain

$$\beta_H = \left(1 - \delta + \frac{\alpha\overline{\lambda}\frac{1+\overline{K^M/K^I}}{\overline{K/Y}} - (\theta\overline{\lambda} - 1)}{\overline{\lambda}(1 + \overline{K^M/K^I}) - 1}\right)^{-1} = 0.9865$$

which is within the range of values used for annual calibration in the literature. Finally, Equation (B1) yields $z^I = 1.0644$. The calibrated parameter values and the calibration targets are summarized in Table 2.

# C Appendix – Extensions

In this Appendix, we discuss four extensions that help to understand the main channel through which shocks get propagated, the robustness of the findings, and the interpretation of the model.

## C.1 Cost of Intermediation

So far, we have assumed that banks do not incur real cost when they monitor entrepreneurs. Typically, however, commercial or universal banks have to spend considerable resources on such activities. Such cost can easily be integrated in our model. Suppose, e.g., that banks incur a cost c ($c > 0$) per unit of loans they monitor. Then, the non-pledgeable part increases to $(c+\theta)K_t^I$, while bankers only obtain $\theta K_t^I$ from their lending activities. The market-imposed leverage constraint, Equation (5), adjusts to

$$K_t^I = \frac{1 + r_t^M}{r_t^M - r_t^I + \theta + c} E_t.$$

The solution of the model follows similar lines as for the baseline model Specifically, similar to Section 5, we get

$$\begin{aligned}
\hat{r}^M &= \delta + \rho_H \\
\theta\hat{\lambda} &= \delta + \rho_B + 1 \\
\hat{\lambda} &= \frac{1 + \delta + \rho_H}{\delta + \rho_H - \hat{r}^I + \theta + c} = \frac{1 + \delta + \rho_B}{\theta} \\
\hat{r}^I &= \hat{r}^M + \frac{\theta(\rho_B - \rho_H)}{1 + \delta + \rho_B} + c.
\end{aligned}$$

Hence, steady state leverage $\hat{\lambda}$ and the return on equity $\theta\hat{\lambda}$ are unaffected by cost of intermediation. However, less capital can be invested in sector $I$ which reduces both the level of bank equity and investor wealth in the steady state.

### C.2 Workers also Save

We consider a variant of the model in which households are not divided into investors and workers, such that there are only two types of agents: households and bankers. The former own the capital stock $\Omega$ and supply labor inelastically. The problem of the household is the same as the one described in Section 3.2, except for the budget constraint (3), which now includes labor income

$$C_t^H + \Omega_{t+1} = w_t^I L^I + w_t^M L^M + \Omega_t(r_t^M + 1 - \delta).$$

Optimization yields the standard Euler equation for the household problem

$$\frac{1}{C_t^H} = \beta_H (1 + r_{t+1}^M - \delta) \frac{1}{C_{t+1}^H}. \tag{C1}$$

The steady state condition derived from the households' necessary condition remains unaffected. As there is no change in the banker's first-order-condition either, the following proposition ensues:

**Proposition 8** (Steady State of Two-Type Economy)**.**
*The steady state in the two-type economy is the same as in the three-type economy.*

*Proof.* The proof directly follows from the previous discussion. □

However, transitional dynamics in the two-agent economy cannot be made explicit, as there are no closed form solutions for the consumption-saving policies. Quantitatively, the results are *mutatis mutandis* of a similar order of magnitude – see Table 10 for wealth shocks.[30]

Table 10: Welfare Cost and Output Multipliers of Wealth Shocks for 2-Agent and 3-Agent Economy

| | multipliers | welfare cost | | |
|---|---|---|---|---|
| | | investor | worker | banker |
| 3-agent model | | | | |
| bank equity shock | 5.0759 | 0.3652 | 0.1696 | 3.3494 |
| investor wealth shock | 4.7639 | 0.4159 | 0.1513 | 0.5338 |
| 2-agent model | | | | |
| bank equity shock | 4.1653 | 0.2084 | | 3.3522 |
| household wealth shock | 3.125 | 0.2041 | | 0.4480 |

*Simulation results for a negative shock of 1 percent of total capital to bank equity or investor wealth. Shocks are at the beginning of the period, i.e. before the production stage. Welfare costs are denominated as the required percentage change of permanent consumption to compensate for the specific shocks.*

## C.3 Anticipated Bank Equity Shocks

We now focus on the impact of anticipated shocks to bank equity. We use a simple but general formulation which can be traced back to particular foundations of such shocks. Suppose that in each period production is deterministic, as in our baseline model, but that the returns of bankers are affected by an additional exogenous shock. Specifically,

[30] A quantitative assessment of the impulse responses to the different shocks is available upon request.

let

$$
r_t^B = \begin{cases} \theta\lambda(r_t^M, r_t^I) - \delta - 1 & \text{with probability } 1-\nu \\ \kappa(\theta\lambda(r_t^M, r_t^I) - \delta) - 1 & \text{with probability } \nu, \end{cases}
$$

where $0 < \kappa < 1$ and $0 < \nu < 1$. The shocks to bank equity are independently and identically distributed across time and realized before the bankers make their consumption-saving decision. Typically for the context of banking crises, $\nu$ is assumed to be rather small and thus such shocks occur infrequently. The bankers' Euler equation reads

$$
\frac{1}{C_t^B} = \beta_B \mathbb{E}_t\left[\frac{1 + r_{t+1}^B}{C_{t+1}^B}\right],
$$

where $\mathbb{E}_t$ denotes the expectation formed at the end of period $t$. As current shocks are realized when the consumption-saving decision is made, the functional form of the saving policy remains unaffected compared to the non-stochastic case. Specifically, $C_t^B = (1-\beta_B)(1 + r_t^B)E_t$ and $E_{t+1} = \beta_B(1 + r_t^B)E_t$, i.e. saving rates remain constant with respect to end-of-period equity. Hence, with high probability, the dynamics associated with anticipated negative shocks to bank equity are approximately the same as if such shocks occurred unanticipated at the deterministic steady state provided that the shocks are sufficiently rare. The reason is that with very high probability, the economy is close to its steady state when a negative bank equity shock occurs.[31]

## C.4 Stochastic Productivity Shocks

In analogy to the real business cycle literature, we introduce stochastic productivity. Typically, we assume

$$
A_t = \overline{A}\mathrm{e}^{a_t}, a_t = \rho a_{t-1} + \eta_t,
$$

where $\eta_t$ is an independent and identically, normally distributed random variable with mean zero and variance $\sigma_\eta^2$. The parameter $\rho$ ($0 < \rho < 1$) measures the persistence of the shock that follows an autoregressive model of order one. By using the same argument as in the previous section, we see that the saving rates with respect to end-of-period wealth of bankers and investors remain unaffected by uncertainty about future returns to capital and bank equity. Hence, the laws of motion essentially remain the same, but with stochastic time paths of all economic variables.

[31] Assuming larger probabilities for negative bank equity shocks would allow to investigate limit distributions and stochastic steady states. For instance, leverage is distributed symmetrically in our calibration if we set $\kappa = 0.8$ and $\nu = 0.2$.

# D Appendix – List of Notations

| parameter/variable | description |
|---|---|
| $i = H, W, B$ | index for investor ($H$), worker($W$), and banker($B$) |
| $j = M, I$ | index for production sector bond financed ($M$) and loan financed ($I$) |
| $E_t$ | bank equity |
| $\Omega_t$ | household wealth |
| $\Gamma$ | partition on state space for which financial frictions are binding |
| $C_t^i$ | consumption of agent $i = H, W, B$ |
| $K_t^j$ | capital input in sector $j = M, I$ |
| $K_t$ | total capital endowment |
| $D_t$ | investment in deposits |
| $B_t$ | investment in bonds |
| $L^j$ | labor input in sector $j = M, I$ |
| $L$ | total labor endowment |
| $l^j$ | share of labor endowment devoted to sector $j = M, I$ |
| $\ell$ | relative labor input $\ell \doteq L^I/L^M$ |
| $r_t^j$ | capital return in sector $j = M, I$ |
| $r_t^D$ | deposit rate |
| $w_t^j$ | wage rate in sector $j = M, I$ |
| $\lambda_t$ | bank leverage |
| $A_t$ | common productivity |
| $z_t^j$ | sector specific productivity in sector $j = M, I$ |
| $z$ | relative productivity $z \doteq (z^I/z^M)^{1/(1-\alpha)}$ |
| $\alpha$ | output elasticity |
| $\delta$ | capital depreciation rate |
| $\theta$ | financial friction |
| $\beta_i$ | discount factor of agent $i = H, W, B$ |
| $\rho_i$ | discount rate of agent $i = H, W, B$ |