The debt capacity of a government

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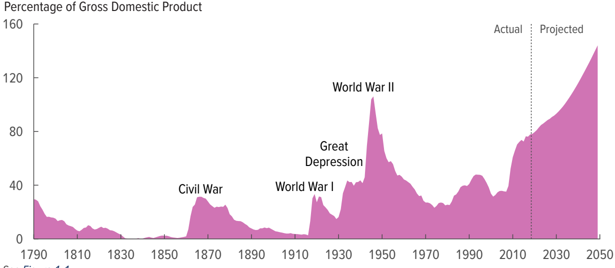
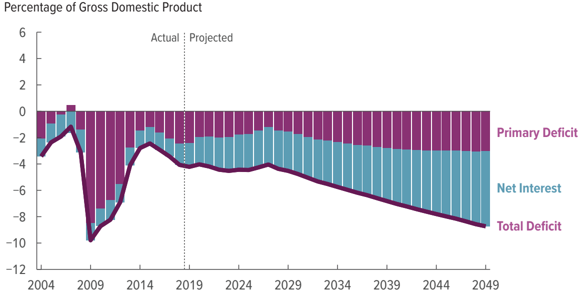
Debt and Deficits



See Figure 1-1



See Figure 1-4

The debt capacity of a government (DEY)

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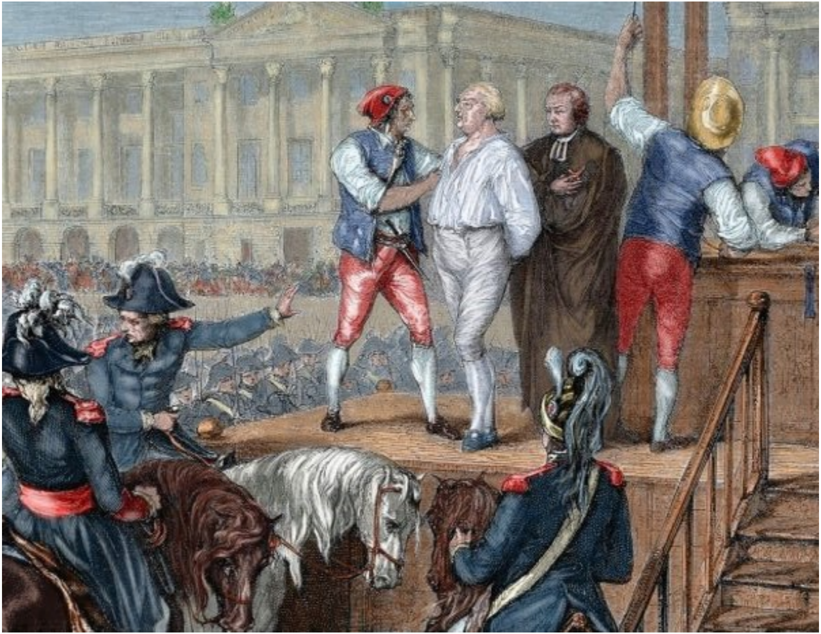

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- In most OECD countries, there is only prospect of government budget deficits in the close to indefinite future.
- The value of their debt cannot be positive! Yet it is.

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  - Chalk (2000) already pointed out that the amount of debt also matters. But he assumed a constant deficit.

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- As a way to study their joint behavior, we endogenize a structural deficit by adding an underfunded social-security scheme financed by debt.
- We ask which level of debt can be sustained.

Outline

- A deterministic model of perpetual refinancing ▸ Model
- The definition of debt capacity ▸ Capacity
- How does it end? ▸ The end ...
- Policy experiments:
  - Too much debt and policy responses ▸ Initial conditions
  - Demographic effects and policy responses ▸ Demography
  - The price level and inflation ▸ Inflation
  - Endogenous growth ▸ Growth
- Conclusions
- (Social security) ▸ Security

A deterministic model of perpetual refinancing

The production function is

Go to Outline

$$Y_t = F\left(K_t, \Lambda_t\right); k_t \triangleq \frac{K_t}{\Lambda_t}; f\left(k_t\right) = F\left(k_t, 1\right)$$

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The households/investors: two-period life. They work at the first point in time only; their supply of labor is inelastic; L^t grows at the constant rate n per period. Generations are born with an endowment of only one kind: their labor force. They collect a wage bill $w_t L_t$.

$$U\left(c_t^t, c_{t+1}^t\right) = u\left(c_t^t\right) + \beta u\left(c_{t+1}^t\right); t \geq 0$$

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The financial market: The one-period rate of return or rate of interest quoted at time t is called r_{t+1}. The young households save an amount $s_t L_t$ at time t. They are indifferent between government debt and the capital stock. In other words,

$$s_t \times L_t \triangleq K_{t+1} + G_{t+1}; s_t \triangleq (1+n)\left(k_{t+1} + g_{t+1}\right); g_t \triangleq G_t / L_t$$

A deterministic model of perpetual refinancing

Taxation and spending:

- Taxation is in the form of a contribution to the social-security system. The time-t young make a total social security contribution of $L_t \tau w_t$, where τ is the social security tax rate.

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Taxation and spending:

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- Government spending is in the form of social-security defined benefits paid to the old households on the basis of the wages they were earning when young. The old receive at time t a total social security benefit of $L_{t-1} \theta w_{t-1}$, where θ is the social security benefit ratio.

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- Throughout we consider the case in which the budget deficit is structural: $\tau < \theta \times (1+n)$.

A deterministic model of perpetual refinancing

With this notation, the simultaneous budget constraints of the households and the government at time t are as follows:

- young household

$$c_t^t + s_t = (1 - \tau)\, w_t$$

- old household

$$c_t^{t-1} = s_{t-1} \times (1 + r_t) + \theta \times w_{t-1}$$

- government

$$-G_{t+1} + \theta \times w_{t-1} \times L_{t-1} = \tau \times w_t \times L_t - (1 + r_t)\, G_t$$

where G_t is the total debt with which the government enters time t and G_{t+1} is the debt with which it exits time t.

Market clearing: The labor market clears

$$\Lambda_t = L_t$$

and the market for goods clears

$$L_t \times c_t^t + L_{t-1} \times c_t^{t-1} + K_{t+1} = F\left(K_t, L_t\right) + \left(1-\delta\right) K_t$$

A deterministic model of perpetual refinancing

Difference equation system:

$$\begin{aligned} s\left(w\left(k_t\right), r\left(k_{t+1}\right)\right) &= (1+n)\left(k_{t+1}+g_{t+1}\right) \\ (1+n)g_{t+1} &= \left(1+r\left(k_t\right)\right)g_t + d\left(k_{t-1}, k_t\right) \end{aligned}$$

where

$$d_t \equiv d\left(k_{t-1}, k_t\right) = \frac{\theta}{1+n} w\left(k_{t-1}\right) - \tau\left(k_t\right) w\left(k_t\right)$$

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Steady state: a situation in which K_t/L_t is a constant k over time. A steady-state must solve the difference equation system with $k_{t+1} = k_t = k_{t-1}$.

- When they exist, there are usually two steady states, one stable and the other unstable.

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\end{aligned}
$$

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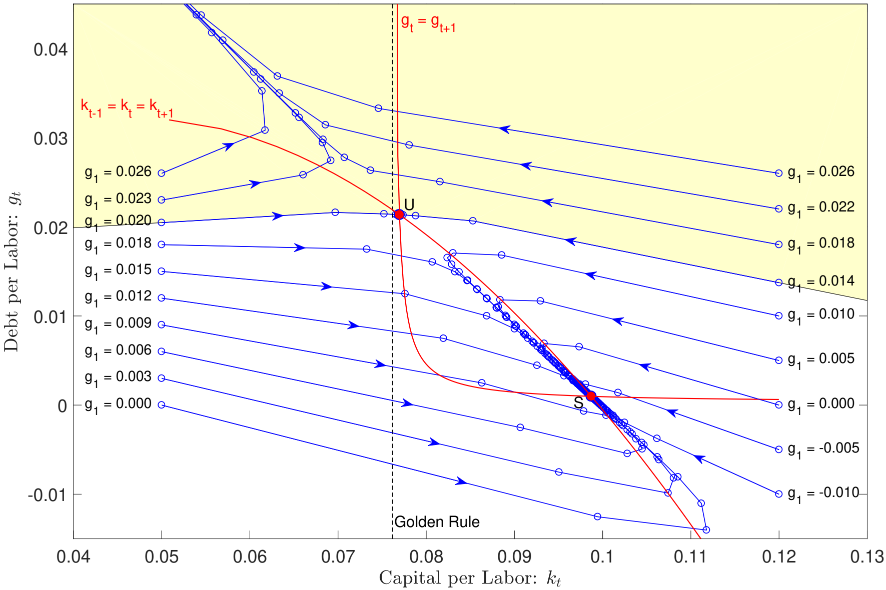
Numerical illustration: log utility and Cobb-Douglas production function. Parameter values are: $n = (1+0.02)^{25} - 1$, $\alpha = 0.2$, $\beta = 0.99^{25}$, $\delta = 1 - (1-0.1)^{25}$, $\theta = 0.165$, $\tau = 0.1$.



$g_t = g_{t+1}$
$k_{t-1} = k_t = k_{t+1}$
U
S
Golden Rule
g_1 = 0.026
g_1 = 0.023
g_1 = 0.020
g_1 = 0.018
g_1 = 0.015
g_1 = 0.012
g_1 = 0.009
g_1 = 0.006
g_1 = 0.003
g_1 = 0.000
g_1 = 0.026
g_1 = 0.022
g_1 = 0.018
g_1 = 0.014
g_1 = 0.010
g_1 = 0.005
g_1 = 0.000
g_1 = -0.005
g_1 = -0.010
Debt per Labor: g_t
Capital per Labor: k_t

A deterministic model of perpetual refinancing

The bubble:

$$g_1 = \underbrace{\frac{1}{N}\sum_{t=1}^{+\infty}\frac{-d_t}{\prod_{\tau=1}^{t}\frac{1+r_\tau}{1+n}}}_{\text{PV of future surpluses}} + \underbrace{\lim_{s\to+\infty}\frac{g_s}{\prod_{\tau=1}^{s-1}\frac{1+r_\tau}{1+n}}}_{\text{Bubble}}$$

At steady state:

$$g = \frac{-d}{r(k)-n}; d = \frac{\theta}{1+n}w(k) - \tau \times w(k)$$

Insight: A bubble is not necessarily explosive. There can exist a perpetual bubble that remains finite.

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The definition of debt capacity

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Debt capacity for a given level of k_0 is the highest level of g_1 such that convergence occurs.

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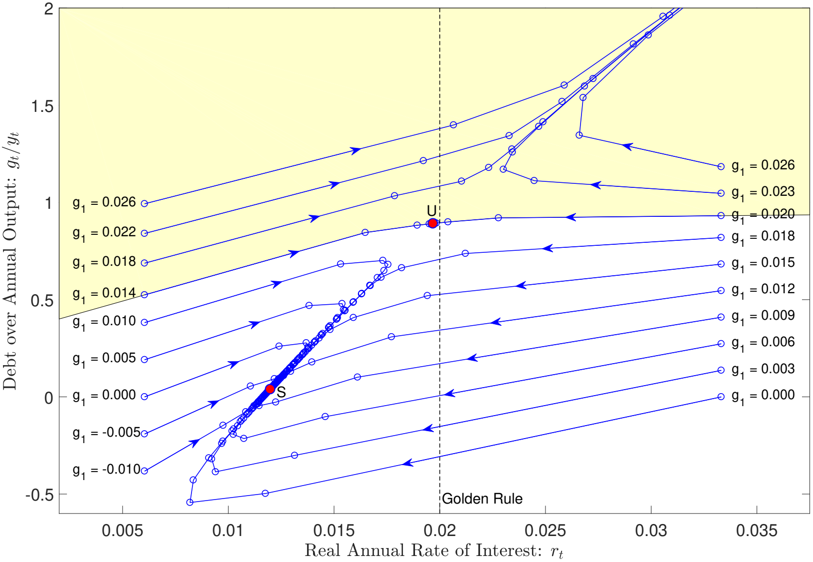
Debt capacity is also the level of debt today that would lead to the unstable steady state along a saddle path. If debt is *strictly* within capacity, it will converge to the stable steady state.



Debt over Annual Output: g_t/y_t
Real Annual Rate of Interest: r_t
Golden Rule
U
S
g_1 = 0.026
g_1 = 0.022
g_1 = 0.018
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g_1 = 0.000
2
1.5
1
0.5
0
-0.5
0.005
0.01
0.015
0.02
0.025
0.03
0.035

How does it end?

- Each household has a two-period horizon and they end their life with zero wealth, as is optimal.

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- As long as the level of debt is strictly below debt capacity,
  - after enough time the debt converges to the stable steady state,
  - which leaves a lot of room for many possible values of the initial real market value of government debt, even without any changes of the social-security rate of contribution τ or rate of benefit θ.
  - They all lead to the same stable ongoing equilibrium with a steady-state debt per capita denoted g_S, and no terminal conditions can be imposed.

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- If the initial face value of debt were ever so slightly above debt capacity,
  - the debt, while finite at any finite future date, could be forecast eventually to explode, and to crowd out the capital stock.
  - This means that explosive paths cannot even begin: the debt cannot be sold, or has zero market value, even at the initial date.

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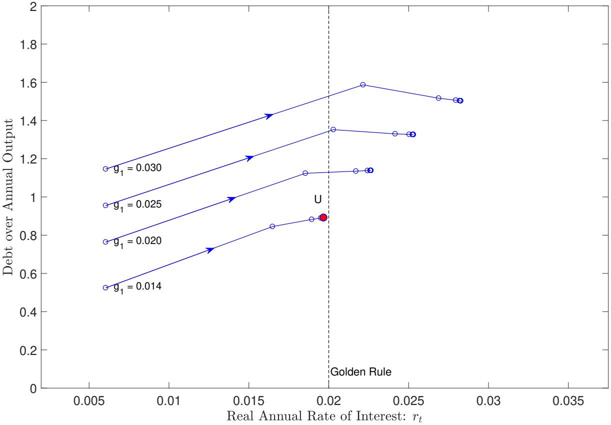
Too high initial debt and policy responses

- ... unless the government increases taxes, or promises to increase taxes, and embarks on a new saddle path ...



Demographics and policy responses

- When at debt capacity (for initial n), a drop in population growth would cause a drop in debt capacity and an explosion

	τ	Steady-state r/year	Steady-state debt /annual output	Steady-state deficit/output
	Initial drop to 1%/year			
$M=0$	0.1288	0.0101	0.6990	-0.0021
$M=1$	0.1339	0.0130	1.0458	-0.104
$M=2$	0.1500	0.0180	1.5215	-0.4275
	Initial drop to 1.5%/year then to 1%/year			
$M=0$	0.1301	0.0111	0.8206	-0.0282
$M=1$	0.1399	0.0152	1.2721	-0.2245
$M=2$	0.1673	0.0218	1.7919	-0.7729

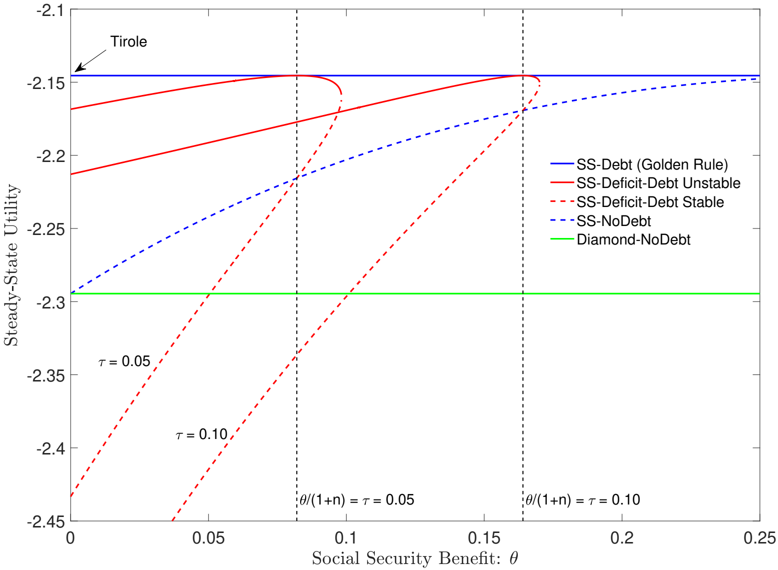
- The price level and inflation ▸ Inflation
- Endogenous growth ▸ Growth
- (Social security) ▸ Security

Conclusion

- The debt of OECD countries today probably contains a rational bubble
- A rational bubble does not necessarily explode
- As long as it is can be determined that it will not explode, we are within debt capacity
- The path of the debt and the deficit must be calculated jointly and forward
- Because of the existence of a stable steady state, there are many possible values for the market value of government debt
- Initial conditions are key: are they within debt capacity?

Social security

- Why Social Security? We consider the case in which the rate of interest is $r < n$.
- Diamond showed that the OLG equilibrium is inefficient ($r < n$) which means that there is too much physical capital.
- Tirole showed that a bubble of just the right size can produce the efficient equilibrium $r = n$ (called the Golden Rule).
- Who can issue such a bubble? Because the government is infinitely lived, it can issue government debt, which, as we saw, can contain a bubble component.
- If the stock of capital is too high, a budget deficit generated by social security is a better use of the proceeds. The figure shows that social security with a steady-state deficit can be a welfare-improving form of spending, relative to the competitive Diamond equilibrium.



-2.1
-2.15
-2.2
-2.25
-2.3
-2.35
-2.4
-2.45
0
0.05
0.1
0.15
0.2
0.25
Steady-State Utility
Social Security Benefit: θ
Tirole
SS-Debt (Golden Rule)
SS-Deficit-Debt Unstable
SS-Deficit-Debt Stable
SS-NoDebt
Diamond-NoDebt
τ = 0.05
τ = 0.10
θ/(1+n) = τ = 0.05
θ/(1+n) = τ = 0.10

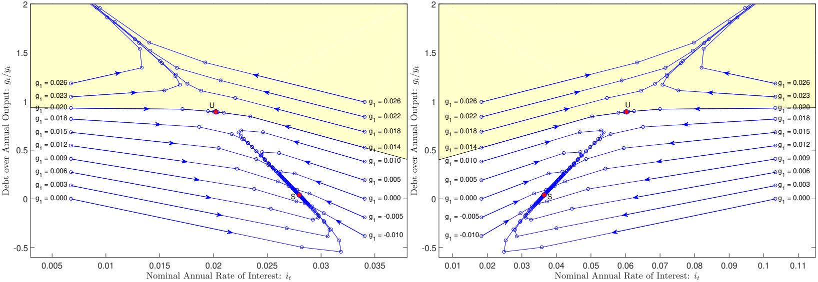
The price level and inflation

- Inescapable connection between government debt and money
- Consider economy with no cash but there exists a price level P_t creating a distinction between real (units of consumption) and nominal quantities
- The outstanding government debt is now contractually denominated in nominal terms (nominal face value)
- In addition to making Social Security payments, the government trades bonds to set the *nominal* rate of interest in order to influence (rationally) anticipated inflation.
- It does that by means of a Taylor rule

$$1 + i_{t+1} = (1 + \bar{\imath}) \times \left(\frac{\frac{P_{t+1}}{P_t}}{1 + \bar{\pi}} \right)^{\phi} ; \phi \geq 0; \phi \neq 1$$

The price level, given by nominal/real values of the debt (FTPL), is indeterminate.

The paths of the debt ratio and the nominal interest rate.
$\phi = 0.5$ (left-hand plot) and $\phi = 1.5$ (right-hand plot).



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Innovation

The production function is

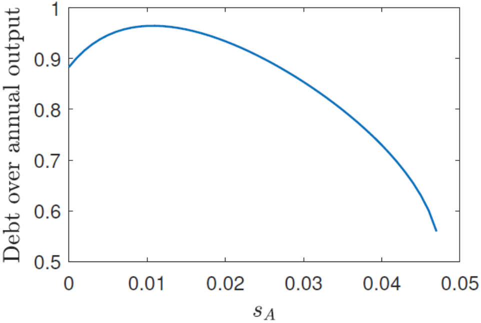
$$Y_t = A_t^{\sigma} F\left(K_t, \Lambda_{Y,t}\right)$$

where $A_t > 0$ is knowledge capital and $0 < \sigma$.
The production and accumulation of knowledge capital: is subsidized by the government $\left(L_{A,t} = s_A L_t\ \right)$. It evolves as

$$A_{t+1} - A_t = \theta_A L_{A,t} A_t^{1-\beta_A}$$

where $\theta_A > 0$ is the productivity of labor in knowledge production and $\beta_A > 0$ captures the extent to which knowledge production becomes more difficult as knowledge accumulates. This is the "modern Romerian" specification of the knowledge production function. See Jones (2019).
The rate of growth: of $k = K/L$ (so far, equal to 0) is called π.

Debt capacity as a function of s_A. Illustration with log utility and Cobb-Douglas production function.



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